

THE DISCRETE MATHIEU EQUATION AT NON REAL COUPLING

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We study the eigenvalues of the discrete Mathieu equation at non real coupling i.e. the eigenvalues of the finite difference operator

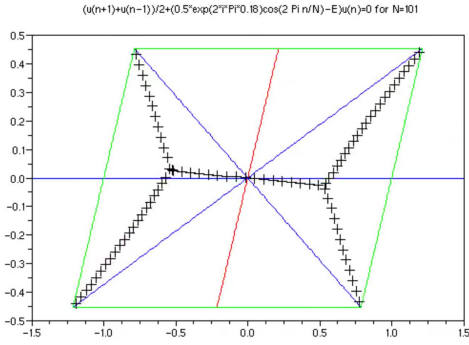
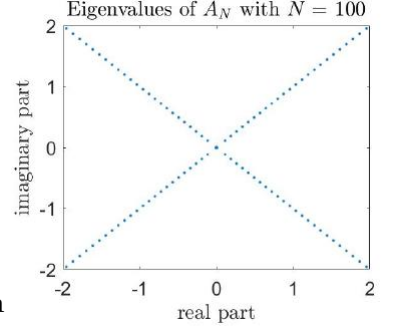
$$(H_{N,\lambda}u)_n = \frac{1}{2}(u_{n+1} + u_{n-1}) + \lambda \cos(2\pi n/N)u_n$$

acting on $\ell^2(\{0, \dots, N-1\})$ with periodic boundary conditions.

The complex parameter λ is assumed not to be real.

When $\lambda = i$, the operator has been dubbed “the Scottish flag operator” because of its spectrum (see figure to the right).

For general non real λ , the spectrum is similar to the one depicted in the figure below.



We will explain how this spectrum can be computed for arbitrary non real λ when N is large. The main tool is a complex WKB method developed for finite difference operators with analytic coefficients that we will present during the talk.

The results were obtained in collaboration with Izak Oltman (Northwestern University).