Sequences

Suppose that c is a constant and the sequences $\{a_n\}$ and $\{b_n\}$ are both convergent. Then

1. $\lim_{n\to\infty}(a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$
2. $\lim_{n\to\infty}(a_n - b_n) = \lim_{n\to\infty} a_n - \lim_{n\to\infty} b_n$
3. $\lim_{n\to\infty} ca_n = c \lim_{n\to\infty} a_n$
4. $\lim_{n\to\infty}(a_n b_n) = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n$
5. $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}$ if $b_n \neq 0$
6. $\lim_{n\to\infty} [a_n]^p = \left[\lim_{n\to\infty} a_n\right]^p$ if $p > 0$ and $a_n > 0$

Other Useful Theorems

- Definition of Convergence (ϵ, N definition)
- If $\lim_{x\to\infty} f(x) = L$ and $f(n) = a_n$ when n is a positive integer, then $\lim_{n\to\infty} a_n = L$.
- $\lim_{n\to\infty} r^n = \begin{cases} 0 & \text{if } |r| < 1 \\ 1 & \text{if } r = 1 \\ \text{diverges} & \text{otherwise} \end{cases}$
- Squeeze Theorem (adapted to sequences)

  If $a_n \leq b_n \leq c_n$ for $n \geq n_0$ and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$, then $\lim_{n\to\infty} b_n = L$.
- If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$.
- If $\lim_{n\to\infty} a_n = L$ and the function f is continuous at L, then $\lim_{n\to\infty} f(a_n) = f(L)$.
- Every bounded, monotonic sequence is convergent.