Math 2260: Calculus II For Science And Engineering

Some Famous Taylor Series

Remember that the *Taylor series* of $f(x)$ with center $x = a$ is

$$P(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$

To get the Taylor polynomial $P_n(x)$, just stop your sum at the nth power.

This table shows some very famous Maclaurin series (center 0) which you should learn. For the first few of these (the ones with radius ∞), you can find them by taking derivatives of $f(x)$ over and over, seeing a pattern, and using that pattern to get $f^{(n)}(0)$. For the last few, you get them by starting with the geometric series $1/(1-x)$, plugging in values for x, and then integrating.

$f(x)$	$P(x)$	=	Pattern of terms	Radius of convergence R
e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!}$	=	$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots$	∞
$\sin(x)$	$\sum_{n=0}^{\infty} (-1)^{n+1} \frac{x^{2n+1}}{(2n+1)!}$	=	$x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots$	∞
$\cos(x)$	$\sum_{n=0}^{\infty} (-1)^{n+1} \frac{x^{2n}}{(2n)!}$	=	$1 - \frac{x^2}{2} + \frac{x^4}{24} - \cdots$	∞
$\frac{1}{1-x}$	$\sum_{n=0}^{\infty} x^n$	=	$1 + x + x^2 + \cdots$	1
$\ln(1+x)$	$\sum_{n=0}^{\infty} (-1)^{n+1} \frac{x^{n+1}}{n+1}$	=	$x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$	1
$\arctan(x)$	$\sum_{n=0}^{\infty} (-1)^{n+1} \frac{x^{2n+1}}{2n+1}$	=	$x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$	1

ONES YOU SHOULD MEMORIZE: e^x, $\sin(x)$, and $1/(1-x)$

Rules of thumb:

- The series with radius ∞ have factorials in the denominator.
- sin has odd terms (it's an odd function), and cos has even terms (it's an even function).
- arctan looks a lot like sin, but the factorials make a big difference!