Integration by Parts, Partial Fractions
Aug 6- PM

Integration by Parts: Integration by parts follows from the product rule for derivatives. You likely saw the formula as

$$\int u\,dv = uv - \int v\,du$$

This is a compact version of the actual formula, but it will suffice.

Example: Evaluate $\int x^2\cos(x)\,dx$. We can let $u = x^2$ and $dv = \cos(x)\,dx$ and then rewrite the integral using the integration by parts formula.

$$\int x^2\cos(x)\,dx = x^2\sin(x) - \int 2x\sin(x)\,dx$$

We need to do integration by parts one more time. Let $u = 2x$ and $dv = \sin(x)\,dx$. Then

$$x^2\sin(x) - \int 2x\sin(x)\,dx = x^2\sin(x) + 2x\cos(x) + \int 2\cos(x)\,dx$$

$$= x^2\sin(x) + 2x\cos(x) - 2\sin(x) + C$$

Partial Fraction Decomposition: Sometimes we want to integrate a rational function. If the denominator is just the right kind of complicated, we can apply partial fraction decomposition which will split the rational function into a sum of simpler rational functions.

Example: Apply partial fraction decomposition to $\frac{3x-9}{(x-1)(x+2)^2}$.

First, we guess at the decomposition. The guessing is restricted to certain rules. In this case, we get

$$\frac{3x-9}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

Equivalently,

$$3x - 9 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

The easiest way to determine A, B, C is called the *cover up method.*

Let $x = 1$, then $-6 = 9A \implies A = -\frac{2}{3}$.

Let $x = -2$, then $-15 = -3C \implies C = 5$.

To find B, plug in a random value for x, say $x = 0$, then
$-9 = 4A - 2B - C \implies B = \frac{2}{3}$