Worksheet - Linear Independence, Bases

Exercise 1. Assume that A is row equivalent to B. Find bases for nul A and col A.

$$A = \begin{bmatrix} 1 & 2 & -5 & 11 & 3 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 5 & -7 & 8 \\ 0 & 0 & 0 & 0 & -9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Exercise 2. True or false? Give brief justifications.

(a) A linearly independent set in a subspace H is a basis for H.

(b) If a finite set S of nonzero vectors spans a vector space V, then some subsets of S is a basis of V.

(c) If B is an echelon form of a matrix A, the pivot columns of B for a basis of col A.

Exercise 3. Find the vector $\mathbf{x}$ determined by the given coordinate vector $[\mathbf{x}]_\beta$ and the given basis β.

$$\beta = \left\{ \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 7 \end{bmatrix} \right\} \qquad [\mathbf{x}]_\beta = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$$

Exercise 4. Find the coordinate vector $[\mathbf{x}]_\beta$ of $\mathbf{x}$ relative to the given basis β.

$$\beta = \left\{ \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \end{bmatrix} \right\} \qquad \mathbf{x} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Exercise 5. Find a basis of the following vector spaces. What is the dimension of each?

$$\left\{ \begin{bmatrix} 4s \\ -3s \\ -t \end{bmatrix} : s, t \text{ in } \mathbb{R} \right\} \qquad \{(a, b, c, d) : a - 3b + c = 0\}$$

Exercise 6. Let $T : V \to W$ be a linear transformation. Shot that if $\{\mathbf{v_1}, \ldots, \mathbf{v_p}\}$ is linearly dependent V, then $\{T(\mathbf{v_1}), \ldots, T(\mathbf{v_p})\}$ is linearly dependent in W. Use this to show that if $\{T(\mathbf{v_1}), \ldots, T(\mathbf{v_p})\}$ is linearly independent in W, then $\{\mathbf{v_1}, \ldots, \mathbf{v_p}\}$ is linearly independent in V.