Exercises: Eigenvalues and Eigenvectors

1–8 ■ Find the eigenvalues of the given matrix.

1. $\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$

2. $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

3. $\begin{bmatrix} 5 & 1 \\ 4 & 5 \end{bmatrix}$

4. $\begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix}$

5. $\begin{bmatrix} 2 & 0 & 0 \\ -1 & 3 & 3 \\ 6 & -6 & -6 \end{bmatrix}$

6. $\begin{bmatrix} 0 & 1 & 2 \\ -4 & 1 & 4 \\ -5 & 1 & 7 \end{bmatrix}$

7. $\begin{bmatrix} 4 & 1 & 6 \\ 0 & 2 & 3 \\ 0 & 0 & 9 \end{bmatrix}$

8. $\begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 5 & 3 & 1 & 1 \\ 4 & 7 & 2 & 2 \end{bmatrix}$

9–12 ■ Find one eigenvector for the given matrix corresponding to the given eigenvalue.

9. $\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$, $\lambda = 5$

10. $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$, $\lambda = -1$

11. $\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$, $\lambda = 0$

12. $\begin{bmatrix} 4 & 1 & 3 \\ 1 & 3 & 1 \\ 2 & 0 & 5 \end{bmatrix}$, $\lambda = 3$

13. Find the eigenvalues of the matrix

$$\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$$

and find one eigenvector for each eigenvalue.

14. The $\lambda = 2$ eigenspace for the matrix

$$\begin{bmatrix} 3 & 4 & 2 \\ 1 & 6 & 2 \\ 1 & 4 & 4 \end{bmatrix}$$

is two-dimensional. Find a basis for this eigenspace.

15. The $\lambda = 1$ eigenspace for the matrix

$$\begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 2 & 1 & 3 \\ 2 & 1 & 6 & 5 \\ 1 & 2 & 4 & 8 \end{bmatrix}$$

is two-dimensional. Find a basis for this eigenspace.

Answers

Note: Any nonzero multiple of an eigenvector is again an eigenvector.

1. $2, 5$ **2.** $3, 4, 7$ **3.** $3, 7$ **4.** $-2, 6$ **5.** $-3, 0, 2$ **6.** $1, 2, 5$ **7.** $2, 4, 9$ **8.** $0, 1, 2, 3$

9. $(1, 2)$ **10.** $(-3, 2)$ **11.** $(-3, 1)$ **12.** $(-1, -2, 1)$ **13.** $\lambda = 1$: $(-2, 1)$, $\lambda = 4$: $(1, 1)$

14. $(-4, 1, 0)$ and $(-2, 0, 1)$ **15.** $(-2, -1, 1, 0)$ and $(-1, -3, 0, 1)$