Worksheet
Aug 13 - AM

Exercise 1. Write $\mathbf{y}$ as the sum of two orthogonal vectors, one in span$\{\mathbf{u}\}$ and one orthogonal to it.

$$\mathbf{y} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \mathbf{u} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

Compute the distance from $\mathbf{y}$ to the line through $\mathbf{u}$ and the origin.

Exercise 2. True and false! Justify your answers!

(a) If A is an $n \times n$ matrix with orthogonal columns, then it is invertible.

(b) If a set $\{\mathbf{u_1}, \ldots \mathbf{u_p}\}$ has the property that $\mathbf{u_i} \cdot \mathbf{u_j} = 0$ whenever $i \neq j$ then S is an orthnormal set.

(c) If c is not 0, then the orthogaonl projection of $\mathbf{y}$ onto a vector $\mathbf{u}$ is the same as the orthogonal projection of $\mathbf{y}$ onto $c\mathbf{u}$.

Exercise 3. Let W be the subspace spanned by the $\mathbf{v}'s$ and write $\mathbf{y}$ as a sum of a vector in W and a vector orthogonal to W.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

What is the closest point in W to $\mathbf{y}$?

Exercise 4. Find an orthogonal basis for $\mathrm{col}(A)$.

$$A = \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$$