

1. **True/False**

- (a) An $n \times n$ matrix that is orthogonal and diagonalizable must be symmetric.
- (b) If $A = A^T$ and if vectors u and v satisfy $Au = 3u$ and $Av = 4v$, then $u \cdot v = 0$.
- (c) An $n \times n$ symmetric matrix has n distinct real eigenvalues.
- (d) If $B = PDP^{-1}$, where $P^T = P^{-1}$ and D is a diagonal matrix, then B is a symmetric matrix.

2. Diagonalize the following symmetric matrix with an orthonormal basis.

$$A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

3. Suppose we know that the matrix

$$A = \begin{pmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{pmatrix}$$

has eigenvalues -2 and 7. Diagonalize A by orthogonal matrices.