Worksheet - Radius of Convergence, Taylor Series

1. Determine whether the following statement is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

   (a) If $\lim_{n \to \infty} a_n = 0$ then $\sum a_n$ converges.

   (b) If $\sum a_n x^n$ diverges when $x = 6$, then it diverges when $x = 10$.

   (c) If $\sum a_n (x + 2)^n$ converges when $x = 0$, then it converges when $x = -1$.

   (d) If $\sum a_n$ is divergent, then $\sum |a_n|$ is divergent.

   (e) If $\sum |a_n|$ is divergent, then $\sum a_n$ is divergent.

   (f) If $a_n > 0$ and $\sum a_n$ converges, then $\sum (-1)^n a_n$ coverges.

   (g) The ratio test can be used to determine whether $\sum 1/n$ converges.

2. Find the interval of convergence of the following series.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n (x - 2)^n}{(n + 2)!}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n^2 5^n}$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{n(x + 1)^n}{4^n}$

3. Find a formula for the given sequence, making sure to give an initial value (eg. $a_n = n$, $n \geq 1$). Then write it as a series $\sum a_n$.

   (a) $1, -x^2, x^4, -x^6, x^8, \ldots$

   (b) $\dfrac{1}{2!} x, \dfrac{2}{3!} x^2, \dfrac{4}{4!} x^3, \dfrac{8}{5!} x^4, \dfrac{16}{6!} x^5, \ldots$

   (c) $-1, \dfrac{2}{5} x, \dfrac{-3}{5^2} x^2, \dfrac{4}{5^3} x^3, \dfrac{-5}{5^4} x^4, \ldots$

   (d) $\dfrac{x^3}{3}, \dfrac{x^5}{5}, \dfrac{x^7}{7}, \dfrac{x^9}{9}, \dfrac{x^{11}}{11} \cdots$

   (e) $x, -x^3, \dfrac{x^5}{2!}, \dfrac{-x^7}{3!}, \dfrac{x^9}{4!}, \cdots$

   (f) $1, x^6, \dfrac{x^{12}}{2}, \dfrac{x^{18}}{3}, \dfrac{x^{24}}{4}, \cdots$

4. Use the power series representation for $\frac{1}{1-x}$ to find power series representations for the following series. Determine the radius of convergence for each new power series.

   (a) $\dfrac{1}{1 + x^2}$

   (b) $\arctan x$

   (c) $\dfrac{1}{1 - 3x}$

   (d) $\ln(1 - 3x)$

   (e) $x^2 \ln(1 - 3x)$

   (f) $\dfrac{1}{(1 - 3x)^2}$

5. Find the Taylor series for the given function centered at the given point.

   (a) $f(x) = \ln(1 + x)$, $a = 0$

   (b) $f(x) = x^4 + 2$, $a = -1$

   (c) $f(x) = 1/x$, $a = -3$

   (d) $f(x) = \sin x$, $a = \pi/2$