Worksheet: Subspaces, Span, and Linear Independence

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(1) Using the subspace test or a graph of the equation, determine if the following subsets of $\mathbb{R}^3$ are subspaces.

(a) $V = \{(x, y, z)^T \in \mathbb{R}^3 \,|\, x^2 + y^2 + z^2 = 1\}$

(b) $V = \{(x, y, z)^T \in \mathbb{R}^3 \,|\, x + 2y + 3z = 0\}$

(c) $V = \{(x, y, z)^T \in \mathbb{R}^3 \,|\, x + y = 0, \text{ and } x - 3z = 0\}$

(2) Use the subspace test to determine if the following are subspaces of the given vector space.

(a) $W = \left\{\text{functions } f : [0, 1] \to \mathbb{R} \,\middle|\, \int_0^1 f(x)\, dx = 0\right\}$ as a subset of integrable functions from $[0, 1]$ to $\mathbb{R}$.

(b) $W = \{\text{polynomials } p(x) \,|\, \deg(p(x)) \leq n, \deg(p(x)) \text{ is odd }\} \cup \{0\}$ as a subset of $\mathcal{P}^n$, the vector space of polynomials of degree $\leq n$.

(3) Show that the vectors $(0,1,0)^T$, $(0,2,1)^T$, and $(-4,3,7)^T$ are linearly independent.

(4) Let $\boldsymbol{v}_1 = (-2,2,2)^T$ and $\boldsymbol{v}_2 = (3,6,0)^T$. Is the vector $\boldsymbol{w} = (-1,4,2)^T$ in $\operatorname{span}(\boldsymbol{v}_1, \boldsymbol{v}_2)$? If so, write $\boldsymbol{w}$ as a linear combination of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$.

(5) What conditions on a, b and c are necessary for $(a,b,c)^T$ to lie in the span of $(-1,3,2)^T$ and $(0,1,1)^T$?