Worksheet - Row, Column, Null Spaces, and Span

1. Find a basis for the row space, column space, and null space of the matrix given below:

$$A = \begin{bmatrix} 3 & 4 & 0 & 7 \\ 1 & -5 & 2 & -2 \\ -1 & 4 & 0 & 3 \\ 1 & -1 & 2 & 2 \end{bmatrix}$$

2. What is the maximum number of linearly independent vectors that can be found in the nullspace of

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 & 1 \\ 2 & 4 & -1 & 5 & 4 \\ 3 & 6 & -1 & 8 & 5 \\ 4 & 8 & -1 & 12 & 8 \end{bmatrix}$$

3. Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear transformation defined by

$$T([x_1, x_2, x_3]) = [2x_1 + 3x_2, x_3, 4x_1 - 2x_2].$$

Find the standard matrix representation of T. Is T invertible? If so, find a formula for T^{-1}.

4. Is the set $S = \{[x, y] \text{ such that } y = x^2\}$ a subspace of $\mathbf{R}^2$?

5. Use the Cauchy-Schwarz inequality

$$|\mathbf{v} \cdot \mathbf{w}| \leq ||\mathbf{v}|| \cdot ||\mathbf{w}||$$

to prove the Triangle Inequality

$$||\mathbf{v} + \mathbf{w}|| \leq ||\mathbf{v}|| + ||\mathbf{w}||.$$

(Hint: Begin by computing $||\mathbf{v} + \mathbf{w}||^2$ with dot products, and then plug in the Cauchy-Schwarz inequality when the opportunity arises.)

6. Determine whether each of the following statements is True or False. No explanation is necessary.

(a) If V is a subspace of $\mathbf{R}^5$ and $V \neq \mathbf{R}^5$, then any set of 5 vectors in V is linearly dependent.

(b) If A is a 4×7 matrix and if the dimension of the nullspace of A is 3, then for any $\mathbf{b}$ in $\mathbf{R}^4$, the linear system $A\mathbf{x} = \mathbf{b}$ has at least one solution.

(c) Any 4 linearly independent vectors in $\mathbf{R}^4$ are a basis for $\mathbf{R}^4$.

(d) If A is an $m \times n$ matrix, then the set of solutions of a linear system $A\mathbf{x} = \mathbf{b}$ must be a linear subspace of $\mathbf{R}^n$.