1. Find the limits:
   a. \( \lim_{n \to \infty} \frac{n(3n+1)^2}{5n^3 + 23n^2 + 10n + 4} \)
   b. \( \lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} \frac{k + 5}{n} \)

2. Compute the integral \( \int_{0}^{5} 3x^2 \, dx \) via the following steps:
   a. Write down a sum which approximates the integral. The sum should use \( n \) rectangles of equal width, and the height of each rectangle should be determined by the right endpoint of the rectangle. (Your answer should be a summation involving the variables \( k \) and \( n \).)
   b. Use the summation formulas to express the summation in part (a) in simpler terms. (This should be a variable expression only involving \( n \).)
   c. Compute the limit as the number of rectangles increases to infinity.

3. Write down a sum which approximates the integral \( \int_{4}^{5} (5x+1) \, dx \) as in 2(a) above.

4. Given that the area of the ellipse \( 30x^2 + y^2 = 30 \) is \( \sqrt{30}\pi \), evaluate the integral \( \int_{0}^{1} \sqrt{30 - 30x^2} \, dx \).

5. Consider the limit \( \lim_{n \to \infty} \frac{4}{n} \sum_{k=1}^{n} \sqrt{4^2 - \left( \frac{k}{n} \right)^2} \).
   a. This limit is obtained by applying the definition of the definite integral to \( \int_{0}^{4} f(x) \, dx \) for what function \( f(x) \)?
   b. Use a graph of \( f(x) \), and geometry, to evaluate this definite integral (and thus, the limit).
1. Find the indefinite integrals:
   a. \( \int (t^3 + 4t^2 - 8t + 3) dt \)
   b. \( \int \frac{9}{\sqrt{t}} dt \)
   c. \( \int \frac{9x^5 + 5x^3 - 4x + 1}{x^3} dx \)

2. Use the fundamental theorem of calculus to find the derivative of \( F(x) \) for
   \( F(x) = \int_2^x \sqrt{t^3 + 5t - 8} dt \).

3. Use the fundamental theorem of calculus to evaluate the definite integrals:
   a. \( \int_0^8 (4x - 7) dx \)
   b. \( \int_1^2 (4x - 7) dx \)
   c. \( \int_1^7 \left( \frac{x+1}{x^4} \right) dx \)

4. Find the value of \( x \) for which \( F(x) = \int_{-8}^x (|t| + 200) dt \) takes its maximum on the interval \([-8, 40]\).
1. Find the definite integrals using the fundamental theorem of calculus. You may need to use a substitution.
   a. $\int_0^x e^t dt$
   b. $\int_0^x (t+3)^2 dt$
   c. $\int_0^x \sqrt{t+9} dt$
   d. $\int_0^x \frac{3}{4t+5} dt$
   e. $\int_0^x 6e^{3t-2} dt$
   f. $\int_0^x 3t^2 e^{t+2} dt$

2. Consider the function $F(x) = \int_{-2}^x \frac{1}{1+t^2} dt$.
   Determine the intervals on which $F(x)$ is increasing.

3. Find the average value of $g(x) = e^{2x}$ on the interval $[1, 4]$.

4. A rock is dropped from a cliff. The velocity of the rock, measured in feet per second, after $t$ seconds, is $v(t) = -32t$. The rock hits the ground 10 seconds later. How high is the cliff?