

MA 665 EXERCISES 5

- (1) Let R be a ring. Prove that every R -module is projective if and only if every R -module is injective.
- (2) Let F be a free $\mathbb{Z}$ -module. Show that $\bigcap_{n=1}^{\infty} nF = 0$. Conclude that, if Q is a nontrivial divisible $\mathbb{Z}$ -module, then Q is not projective.
- (3) Let M_1 and M_2 be R -modules. Show that $M_1 \oplus M_2$ is an injective R -module if and only if both M_1 and M_2 are injective R -modules. Conclude that, if R is a PID that is not a field, then no nonzero finitely generated R -module is injective.