

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

- | | |
|--|--|
| 3. (a) (b) (c) (d) <input checked="" type="radio"/> (e) | 12. (a) (b) <input checked="" type="radio"/> (c) (d) (e) |
| 4. (a) (b) (c) (d) <input checked="" type="radio"/> (e) | 13. <input checked="" type="radio"/> (a) (b) (c) (d) (e) |
| 5. (a) (b) (c) <input checked="" type="radio"/> (d) (e) | 14. (a) (b) (c) (d) <input checked="" type="radio"/> (e) |
| 6. (a) <input checked="" type="radio"/> (b) (c) (d) (e) | 15. (a) (b) (c) (d) <input checked="" type="radio"/> (e) |
| 7. (a) <input checked="" type="radio"/> (b) (c) (d) (e) | 16. (a) (b) <input checked="" type="radio"/> (c) (d) (e) |
| 8. <input checked="" type="radio"/> (a) (b) (c) (d) (e) | 17. <input checked="" type="radio"/> (a) (b) (c) (d) (e) |
| 9. (a) (b) (c) <input checked="" type="radio"/> (d) (e) | 18. (a) (b) (c) <input checked="" type="radio"/> (d) (e) |
| 10. (a) <input checked="" type="radio"/> (b) (c) (d) (e) | 19. (a) <input checked="" type="radio"/> (b) (c) (d) (e) |
| 11. (a) (b) <input checked="" type="radio"/> (c) (d) (e) | 20. (a) (b) <input checked="" type="radio"/> (c) (d) (e) |

For grading use:

Multiple Choice	Short Answer
(number right)	(out of 10 points)
(5 points each)	

Total	
	(out of 100 points)

Fall 2015 Exam 1 Short Answer Questions

Write answers on this page. You must show appropriate legible work to be sure you will get full credit.

- 3 pts 1. Find the average rate of change of $f(x) = \sqrt{3x+8}$ from $x=1$ to $x=3$.

You do NOT need to simplify your answer.

$$\begin{aligned} \text{AROC} &= \frac{f(b) - f(a)}{b - a} = \frac{f(3) - f(1)}{3 - 1} \\ &= \frac{\sqrt{3(3)+8} - \sqrt{3(1)+8}}{2} = \boxed{\frac{\sqrt{17} - \sqrt{11}}{2}} \end{aligned}$$

- 7 pts 2. Use the limit definition of the derivative to find $f'(x)$ for $f(x) = 11x^2 + 2$.

You must use the limit definition to get credit. Show work clearly.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{(x+h) - x} \\ &= \lim_{h \rightarrow 0} \frac{[11(x+h)^2 + 2] - [11x^2 + 2]}{h} \\ &= \lim_{h \rightarrow 0} \frac{11(x^2 + 2xh + h^2) + 2 - 11x^2 - 2}{h} \\ &= \lim_{h \rightarrow 0} \frac{11x^2 + 22xh + 11h^2 + 2 - 11x^2 - 2}{h} \\ &= \lim_{h \rightarrow 0} \frac{22xh + 11h^2}{h} = \lim_{h \rightarrow 0} \frac{h(22x + 11h)}{h} \\ &= \lim_{h \rightarrow 0} 22x + 11h = 22x + 11(0) = \boxed{22x} \end{aligned}$$

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

3. Solve the equation $6x^2 + 102xy + 4y = 5$ for y in terms of x

Possibilities:

(a) $y = \frac{-102 \pm \sqrt{10308}}{12}$

(b) $y = \frac{5 - 6x^2 - 102x}{4}$

(c) $y = \frac{102x + 4}{6x^2 - 5}$

(d) $y = \frac{6x^2 - 5}{102x + 4}$

(e) $y = \frac{5 - 6x^2}{102x + 4}$

$$\begin{aligned} 6x^2 + 102xy + 4y &= 5 \\ 102xy + 4y &= 5 - 6x^2 \\ y(102x + 4) &= 5 - 6x^2 \\ y &= \frac{5 - 6x^2}{102x + 4} \end{aligned}$$

4. Evaluate $f(10)$ when $f(x)$ is given by the piecewise definition

$$f(x) = \begin{cases} x^2 - 6 & \text{if } x \leq 4 \\ x - 9 & \text{if } 4 < x \leq 8 \\ x^2 - 3x & \text{if } 8 < x \end{cases}$$

↖ 10

Possibilities:

(a) -1

(b) 1

(c) DNE

(d) 94

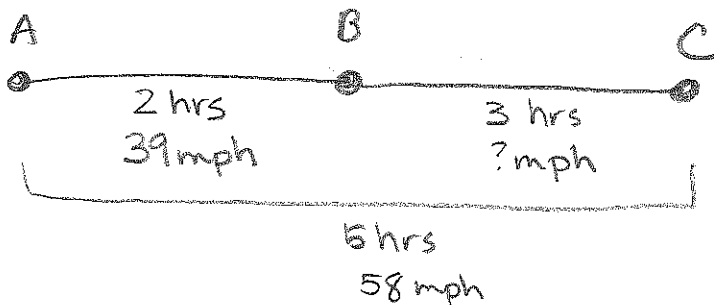
(e) 70

$$\begin{aligned} f(10) &= 10^2 - 3(10) \\ &= 100 - 30 = 70 \end{aligned}$$

5. A train travels from city A to city B, and then travels from city B to city C. The train leaves city A at time 9:00 am and arrives at city B at 11:00 am. The train leaves city B at 11:00 am and arrives at city C at 2:00 pm. The average velocity of the train while traveling from A to B, was 39 miles per hour. What was the average velocity of the train from city B to city C, given that the average velocity of the train while traveling from A to C, was 58 miles per hour?

Possibilities:

- (a) 97 miles per hour
 (b) $(63/2)$ miles per hour
 (c) $(97/3)$ miles per hour
 (d) $(212/3)$ miles per hour
 (e) $(19/2)$ miles per hour



from A \rightarrow B

$$d = rt = (39)(2) = 78 \text{ miles}$$

from A \rightarrow C

$$d = rt = (58)(5) = 290 \text{ miles}$$

then B \rightarrow C is $290 - 78 = 212$ miles average velocity = $\frac{\text{distance}}{\text{time}} = \frac{212}{3} = \frac{212}{3} \text{ mph}$

6. If $f(x) = \frac{4}{x+7}$ then choose the simplified form of $\frac{f(x+h)-f(x)}{h}$:

Possibilities:

- (a) $\frac{8x + 56 + 4h}{(x+h+7)(x+7)(2x+h)}$
 (b) $-\frac{4}{(x+h+7)(x+7)}$
 (c) $\frac{4}{(x+h+7)(x+7)}$
 (d) $-\frac{4}{(x+h+7)^2}$
 (e) $\frac{hx^2 + 14hx + 49h - 4}{(x+7)^2}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{4}{x+h+7} - \frac{4}{x+7}}{h}$$

multiply top & bottom by $(x+h+7)(x+7)$

$$= \frac{4(x+7) - 4(x+h+7)}{(x+h+7)(x+7)(h)}$$

$$= \frac{4x + 28 - 4x - 4h - 28}{(x+h+7)(x+7)(h)}$$

$$= \frac{-4h}{(x+h+7)(x+7)(h)}$$

$$= \frac{-4}{(x+h+7)(x+7)}$$

7. Find a value of x so that the instantaneous rate of change of $f(x) = 4x^2 + 8$ at x is equal to 56.

Possibilities:

- (a) $x = 6$
- (b) $x = 7$
- (c) $x = 8$
- (d) $x = 9$
- (e) $x = 10$

IRoC = the derivative

for $p(x) = Ax^2 + Bx + C$, then $p'(x) = 2Ax + B$

$$\begin{aligned} \text{then if } f(x) &= 4x^2 + 8 \\ &= 4x^2 + 0x + 8 \end{aligned}$$

$$f'(x) = 2(4)x + 0 = 8x$$

$$8x = 56$$

$$x = 7$$

8. For the function $f(x) = 6x^2 + 7x + 3$, find the equation of the tangent line to graph of f at $x = 5$.

Possibilities:

- (a) $y = 67x - 147$
- (b) $y = x^3 + 17$
- (c) $y = 67x + 188$
- (d) $y = 188x - 1017$
- (e) $y = 188$

Slope of tangent line = $f'(x)$

$$\begin{aligned} \text{Using rule above, } f'(x) &= 2(6)x + 7 \\ &= 12x + 7 \end{aligned}$$

$$\text{at } x=5, f'(5) = 12(5) + 7 = 67$$

So slope is 67. Now we need a point.

$$f(5) = 6(5^2) + 7(5) + 3 = 188 \quad (5, 188)$$

Using Point-slope form,

$$y - 188 = 67(x - 5)$$

$$y - 188 = 67x - 335$$

$$y = 67x - 147$$

9. If $\lim_{x \rightarrow 3} f(x) = 11$ and $\lim_{x \rightarrow 3} g(x) = 17$, then what is the value of $\lim_{x \rightarrow 3} \frac{(x+5)(f(x)+1)}{g(x)}$?

Possibilities:

- (a) $\frac{11}{17}$
- (b) 0
- (c) the limit is infinity or does not exist

(d) $\frac{(3+5)(11+1)}{17}$

(e) $\frac{(3)(11)}{17}$

$$\lim_{x \rightarrow 3} \frac{(x+5)(f(x)+1)}{g(x)}$$

$$= \frac{\left(\lim_{x \rightarrow 3} (x+5) \right) \left(\lim_{x \rightarrow 3} (f(x)+1) \right)}{\lim_{x \rightarrow 3} g(x)}$$

$$\lim_{x \rightarrow 3} g(x)$$

$$= \frac{(3+5)(11+1)}{17} = \frac{15 \cdot 12}{17} = \frac{180}{17}$$

10. Find the limit

$$\lim_{t \rightarrow 0^+} \frac{36t^2}{t}$$

"0/0"

Possibilities:

(a) 18

(b) 0

(c) 36

(d) $\frac{18}{\sqrt{t}}$

(e) This limit either tends to infinity or this limit fails to exist

$$\lim_{t \rightarrow 0^+} \frac{36t^2}{t} = \lim_{t \rightarrow 0^+} 36t = 36(0) = \boxed{0}$$

11. Find the limit

$$\lim_{x \rightarrow 0} \left(\frac{15}{x} + \frac{6x - 15}{x} \right)$$

$$= \lim_{x \rightarrow 0} \frac{6x}{x} = \lim_{x \rightarrow 0} 6 = 6$$

Possibilities:

(a) 1

(b) 0

(c) 6

(d) 15

(e) This limit does not exist.

12. Find the limit

$$\lim_{n \rightarrow \infty} \frac{(n+3)^2}{13n+11}$$

$$= \lim_{n \rightarrow \infty} \frac{n^2 + 6n + 9}{13n + 11}$$

distribute

$$= \lim_{n \rightarrow \infty} \frac{n^2}{13n}$$

Since $n \rightarrow \infty$
only terms with
highest order matter

$$= \lim_{n \rightarrow \infty} \frac{n}{13}$$

$$= \frac{\infty}{13} \text{ unbounded}$$

Possibilities:

(a) $\frac{1}{11}$

(b) $\frac{1}{24}$

(c) The limit does not exist or approaches infinity

(d) $\frac{1}{13}$

(e) $\frac{9}{13}$

13. For the function

$$f(x) = \begin{cases} |4 + 8x| & \text{if } x < -2 \\ \sqrt{x^2 + 6} & \text{if } -2 \leq x < 3 \\ 3x^2 + x + 5 & \text{if } 3 \leq x \end{cases}$$

find $\lim_{x \rightarrow 5^+} f(x)$

Possibilities:

(a) 85

(b) $\sqrt{15}$

(c) $\sqrt{31}$

(d) 35

(e) 44

$$\begin{aligned} \lim_{x \rightarrow 5^+} f(x) &= \lim_{x \rightarrow 5^+} (3x^2 + x + 5) \\ &= 3(5)^2 + (5) + 5 \\ &= 75 + 5 + 5 \\ &= 85 \end{aligned}$$

$5 \geq 3$ so we're considering the 3rd line

14. The graph of $y = f(x)$ is shown below. Compute $\lim_{x \rightarrow 1^-} f(x)$.

Possibilities:

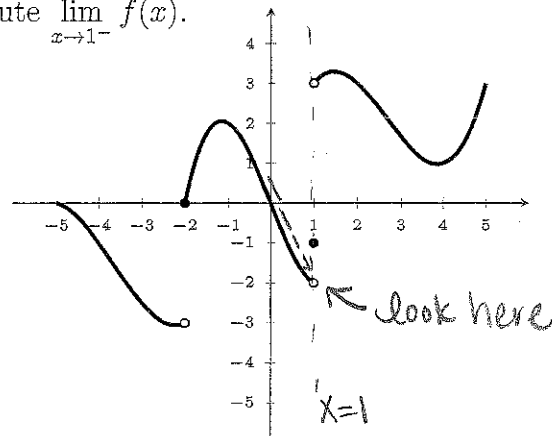
(a) 3

(b) 0

(c) -1

(d) -3

(e) -2



$\lim_{x \rightarrow 1^-} f(x)$ means we look at the function value as we approach 1 from the left (values smaller than 1)

15. Consider the function $f(x) = \begin{cases} x^2 - 4 & \text{if } x < 8 \\ 2x + B & \text{if } x \geq 8 \end{cases}$

Find a value of B so that the function is continuous at $x = 8$.

Possibilities:

- (a) 40
- (b) 41
- (c) 42
- (d) 43
- (e) 44

To be continuous at $x=8$ then

$\lim_{x \rightarrow 8} f(x)$ exists thus both one-sided limits must be equal.

$$\lim_{x \rightarrow 8^-} f(x) = \lim_{x \rightarrow 8^-} (x^2 - 4) = 8^2 - 4 = 60 \quad \leftarrow \text{equal}$$

$$\lim_{x \rightarrow 8^+} f(x) = \lim_{x \rightarrow 8^+} (2x + B) = 2(8) + B = 16 + B$$

$$\begin{aligned} \text{So } 60 &= 16 + B \\ 44 &= B \end{aligned}$$

16. Find the value of m which makes $f(x)$ differentiable everywhere, where

$$f(x) = \begin{cases} x^2, & \text{if } x \leq 4; \\ m(-4 + x) + 16, & \text{if } x > 4 \end{cases}$$

Possibilities:

- (a) 6
- (b) 7
- (c) 8
- (d) 9
- (e) 10

To be differentiable at $x=4$, the function values and the derivative values must match from the left and right.

function values: $\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^-} x^2 = 4^2 = 16$

$$\lim_{x \rightarrow 4^+} f(x) = \lim_{x \rightarrow 4^+} m(-4 + x) + 16 = m(-4 + 4) + 16 = 16$$

these match no matter what m is ✓

derivative values:

$$\begin{aligned} \text{for } x \leq 4 & \quad f'(x) = 2x \\ \text{for } x > 4 & \quad f'(x) = m \end{aligned} \quad \left. \begin{array}{l} \text{using formulas we} \\ \text{learned in Ch 2 for} \\ \text{lines and quadratics} \end{array} \right\}$$

at $x=4$ these should be equal

$$\begin{aligned} 2x &= m \\ 2(4) &= m \end{aligned} \quad \text{m} = 8$$

17. Find the equation of the tangent line to the graph of the function $f(x) = \frac{1}{x^2+1} + 4$ at $x = 3$. You

may use $f'(x) = -\frac{2x}{(x^2+1)^2}$

We need a point and a slope.

Point: $f(3) = \frac{1}{3^2+1} + 4 = \frac{1}{10} + 4 = \frac{41}{10}$
 $(3, \frac{41}{10})$

Possibilities:

(a) $y = -\frac{3}{50}x + \frac{107}{25}$

(b) $y = x^3 + 17$

(c) $y = \frac{41}{10}x - \frac{309}{25}$

(d) $y = -\frac{3}{50}x + \frac{41}{10}$

(e) $y = \frac{41}{10}$

Slope: (the derivative)

we're given $f'(x) = \frac{-2x}{(x^2+1)^2}$

then $f'(3) = \frac{-2(3)}{(3^2+1)^2} = \frac{-6}{100} = \frac{-3}{50}$

Use point-slope form

$$y - \frac{41}{10} = \frac{-3}{50}(x - 3)$$

$$y - \frac{41}{10} = \frac{-3}{50}x + \frac{9}{50}$$

$$y = \frac{-3}{50}x + \frac{214}{50}$$

18. Consider the function $f(x) = 7x^2 + 8x + 4$. Its tangent line at $x = 5$ goes through the point $(9, y_1)$ where y_1 is:

Possibilities:

(a) 219

(b) -171

(c) 78

(d) 531

(e) 134

find the equation of the tangent line.

at $x=5$, $f(5) = 7(5^2) + 8(5) + 4 = 219$

slope comes from derivative

$f'(x) = 2(7)x + 8$ (using formula we know)
 $= 14x + 8$

thus $f'(5) = 14(5) + 8 = 78$

then use point-slope form:

$$y - 219 = 78(x - 5)$$

$$y - 219 = 78x - 390$$

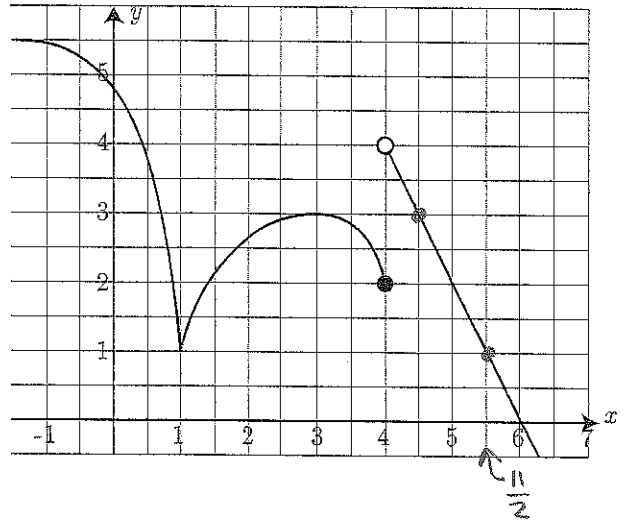
$$y = 78x - 171$$

Now if the point on the line has x value 9 $\rightarrow y = 78(9) - 171 = 531$

19. The graph of $y = f(x)$ is shown below. $f'(\frac{11}{2})$ is approximately :

Possibilities:

- (a) The limit does not exist or tends to infinity
- (b) -2
- (c) 2
- (d) $-\frac{1}{2}$
- (e) $\frac{1}{2}$



$f'(\frac{11}{2})$ is the Slope of the tangent line to the point $x = \frac{11}{2}$

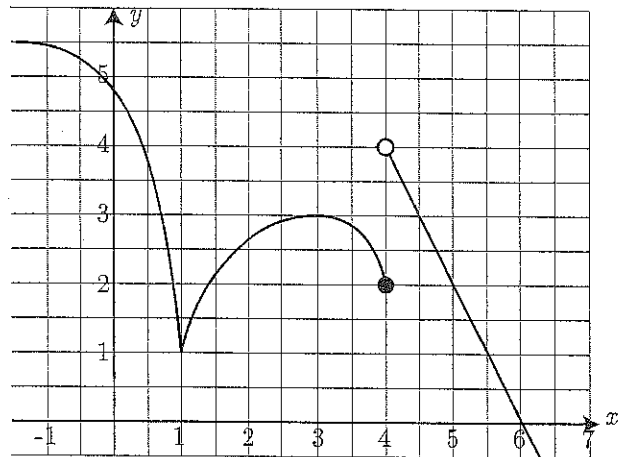
Since its a straight line function on this part of the graph pick two points $(\frac{9}{2}, 3)$ and $(\frac{11}{2}, 1)$

$$m = \frac{\Delta y}{\Delta x} = \frac{1-3}{\frac{11}{2}-\frac{9}{2}} = \frac{-2}{1} = -2$$

20. The graph of $y = f(x)$ is shown below. The function is continuous, except at $x =$

Possibilities:

- (a) $x=1$, $x=3$, and $x=4$
- (b) $x=1$ only
- (c) $x=4$ only
- (d) $x=1$ and $x=4$
- (e) $x=1$, $x=3$, $x=4$, and $x=6$



look for holes, jumps,
or skips in the graph
such as $x=4$

