MA123 — Elem. Calculus Fall 2016
Exam 1 2016-09-22

Name: _____ Sec.: _____

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name on this page, and at the top of page three.

**GOOD LUCK!**

3. (a) (b) (c) (d) (e)
4. (a) (b) (c) (d) (e)
5. (a) (b) (c) (d) (e)
6. (a) (b) (c) (d) (e)
7. (a) (b) (c) (d) (e)
8. (a) (b) (c) (d) (e)
9. (a) (b) (c) (d) (e)
10. (a) (b) (c) (d) (e)
11. (a) (b) (c) (d) (e)
12. (a) (b) (c) (d) (e)
13. (a) (b) (c) (d) (e)
14. (a) (b) (c) (d) (e)
15. (a) (b) (c) (d) (e)
16. (a) (b) (c) (d) (e)
17. (a) (b) (c) (d) (e)
18. (a) (b) (c) (d) (e)
19. (a) (b) (c) (d) (e)
20. (a) (b) (c) (d) (e)

For grading use:

| Multiple Choice | Short Answer |
|---|---|
| (number right) (5 points each) | (out of 10 points) |

| Total | |
|---|---|
| | (out of 100 points) |

Name: Last 4 digits of Student ID:

**Fall 2016 Exam 1 Short Answer Questions**

*Write answers on this page. You must show appropriate legible steps to be sure you will get full credit.*

1. Evaluate the limit $\lim_{x\to 3}\frac{x^2-2x-3}{x^2-3x}$.

Plug in $x=3 \Rightarrow \frac{3^2-2(3)-3}{3^2-3(3)} = \frac{0}{0}$ ⇐ Do more work

$$\lim_{x\to 3}\frac{x^2-2x-3}{x^2-3x} = \lim_{x\to 3}\frac{(x+1)(x-3)}{x(x-3)} = \lim_{x\to 3}\frac{x+1}{x} \quad \text{plug in } x=3$$

$$= \frac{3+1}{3}$$

$$= \boxed{\frac{4}{3}}$$

2. Let $f(x)=3x^2+10x-4$. Find a value of $x$ such that the **instantaneous rate of change** of $f(x)$ at $x$ equals 28.

Instantaneous rate of change of f(x) at x is $f'(x)$.

$f'(x) = 6x+10$ (since the derivative of $ax^2+bx+c$ is $2ax+b$)

Set $6x+10=28$ and solve

$6x=18$

$\boxed{x=3}$

Name: _____

## Multiple Choice Questions

*Show all your work on the page where the question appears.*
*Clearly mark your answer both on the cover page on this exam and in the corresponding questions that follow.*

3. Solve the equation $t = r + \frac{k}{5}w$ for $w$.

**Possibilities:**

(a) $w = \frac{5t - 5r}{k}$

(b) $w = \frac{5r - 5t}{k}$

(c) $w = \frac{t}{r + \frac{k}{5}}$

(d) $w = \frac{5t}{r + k}$

(e) $w = \frac{k}{5t - 5r}$

$t = r + \frac{k}{5}w$ subtract $r$

$t - r = \frac{k}{5}w$ Multiply by 5

$5t - 5r = k \cdot w$ Divide by $k$

$\boxed{\frac{5t - 5r}{k} = w}$

4. Evaluate $f(4)$ when $f(x)$ is given by the piecewise definition

$$f(x) = \begin{cases} x^2 - 6 & \text{if } x \leq 2 \\ 7x - 3 & \text{if } 2 < x \leq 4 \\ x^2 - 9x & \text{if } 4 < x \end{cases}$$

← plug $x=4$ into this piece

**Possibilities:**

(a) $-20$

(b) 10

(c) 15

(d) DNE

(e) 25

$7(4) - 3 = 28 - 3 = \boxed{25}$

5. If $h(t)$ represents the height of an object in feet above ground level at time $t$ seconds and $h(t)$ is given by $h(t) = -16t^2 + 21t + 157$, find the time at which the speed of the object is zero.

**Possibilities:**

(a) (21/16) seconds

(b) (157/32) seconds

(c) (21/32) seconds

(d) (53/32) seconds

(e) 157 seconds

The speed of the object is zero when the derivative $h'(t) = 0$

$h'(t) = -32t + 21 = 0$

$\Rightarrow -32t = -21$

$\Rightarrow$ $\boxed{t = \frac{21}{32} \text{ seconds}}$

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6. If $f(x) = \dfrac{4}{x+8}$ then choose the simplified form of $\frac{f(x+h)-f(x)}{h}$:

**Possibilities:**

(a) $\dfrac{4}{(x+h+8)(x+8)}$

(b) $\dfrac{hx^2 + 16hx + 64h - 4}{(x+8)^2}$

(c) $-\dfrac{4}{(x+h+8)(x+8)}$

(d) $-\dfrac{4}{(x+h+8)^2}$

(e) $\dfrac{8x + 64 + 4h}{(x+h+8)(x+8)(2x+h)}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{4}{x+h+8} - \frac{4}{x+8}}{h}$$

$$= \frac{\frac{(x+8)}{(x+8)} \cdot \frac{4}{(x+h+8)} - \frac{4}{(x+8)} \cdot \frac{(x+h+8)}{(x+h+8)}}{h}$$

$$= \frac{4x+32 - (4x+4h+32)}{(x+8)(x+h+8)} \cdot \frac{1}{h}$$

$$= \frac{-4\cancel{h}}{(x+8)(x+h+8)\cancel{h}}$$

$$= \boxed{\frac{-4}{(x+h)(x+h+8)}}$$

7. Let $f(x) = 4x^2 + 2x + 12$. Find the slope of the tangent line to the graph of $y = f(x)$ at $x = 1$.

**Possibilities:**

(a) $m = 7$
(b) $m = 8$
(c) $m = 9$
(d) $m = 10$
(e) $m = 11$

Slope of tangent line $= f'(1)$

$f'(x) = 8x + 2$

$\Rightarrow f'(1) = 8(1) + 2 = \boxed{10}$

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8. Let $f(x) = x^2 + 2x + 4$. Find a value $c$ between $x = 2$ and $x = 6$, so that the average rate of change of $f(x)$ from $x = 2$ to $x = 6$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$.

**Possibilities:**

(a) 4
(b) 5
(c) 6
(d) 7
(e) 8

Average rate of change $= \dfrac{f(6) - f(2)}{6 - 2}$

$= \dfrac{6^2 + 2(6) + 4 - (2^2 + 2(2) + 4)}{4}$

$= 10$

Instantaneous rate of change at $x = c = f'(c)$

$f'(c) = 2c + 2$ set $2c + 2 = 10$ and solve

$\boxed{c = 4}$

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9. If $\lim_{x\to 17} f(x) = 11$ and $\lim_{x\to 17} g(x) = 5$, then what is the value of $\lim_{x\to 17} \dfrac{3f(x)+2}{x+g(x)}$?

**Possibilities:**

(a) $\dfrac{11}{5}$

(b) $\dfrac{(3)(11)(17)+2}{17+(5)(17)}$

(c) $\dfrac{(3)11+2}{17+5}$

(d) the limit is infinity or does not exist

(e) 0

$$\lim_{x\to 17} \frac{3\cdot f(x)+2}{x+g(x)} = \frac{3\cdot \lim_{x\to 17} f(x) + 2}{\lim_{x\to 17} x + \lim_{x\to 17} g(x)} = \boxed{\frac{3(11)+2}{17+5}}$$

10. Find the limit

$$\lim_{x\to 36} \frac{x^2-9}{x-36}$$

**Possibilities:**

(a) This limit either tends to infinity or this limit fails to exist

(b) 1

(c) 0

(d) $\dfrac{9}{36}$

(e) 39

plug in $x = 36$ to get $\dfrac{36^2-9}{36-36} = \dfrac{1287}{0}$

$\Rightarrow$ limit does not exist.

11. Find the one-sided limit

$$\lim_{t\to 0^-} \frac{|36t|}{t}$$

$$= 36 \cdot \lim_{t\to 0^-} \frac{|t|}{t} = 36(-1) = \boxed{-36}$$

**Possibilities:**

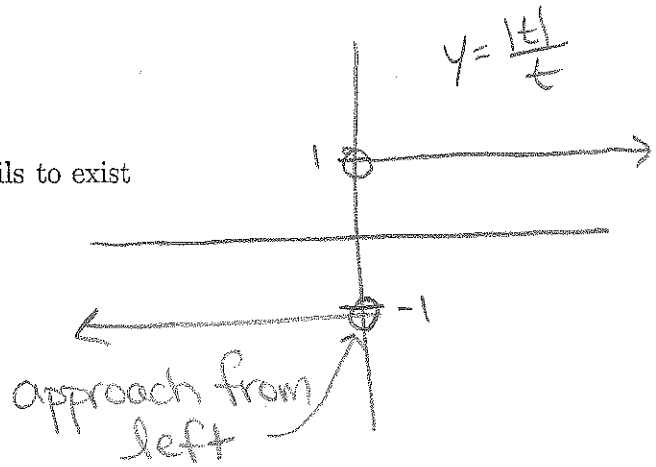
(a) 0

(b) -36

(c) 36

(d) $\frac{18}{\sqrt{t}}$

(e) This limit either tends to infinity or this limit fails to exist




12. Find the limit

$$\lim_{n\to\infty} \frac{(8n+3)^2}{13n^2+7}$$

**Possibilities:**

(a) $\frac{8}{13}$

(b) $\frac{9}{7}$

(c) $\frac{64}{13}$

(d) $\frac{64}{7}$

(e) The limit does not exist or approaches infinity

$$= \lim_{n\to\infty} \frac{64n^2+48n+9}{13n^2+7}$$ only need leading terms

$$= \lim_{n\to\infty} \frac{64n^2}{13n^2}$$

$$= \lim_{n\to\infty} \frac{64}{13} = \boxed{\frac{64}{13}}$$

13. For the function

$$f(x) = \begin{cases} |4+2x| & \text{if } x < -1 \\ \sqrt{x^2+8} & \text{if } -1 \le x < 3 \\ 8x^2+4x+1 & \text{if } 3 \le x \end{cases}$$

use this piece

find $\lim_{x\to 6^+} f(x)$

**Possibilities:**

(a) 85

(b) $\sqrt{17}$

(c) $\sqrt{44}$

(d) 313

(e) 16

$$\lim_{x\to 6^+} 8x^2+4x+1$$

$$= 8(6)^2+4(6)+1$$

$$= 313$$

14. The graph of $y = f(x)$ is shown below. Compute $\lim_{x\to 2^-} f(x)$.

**Possibilities:**

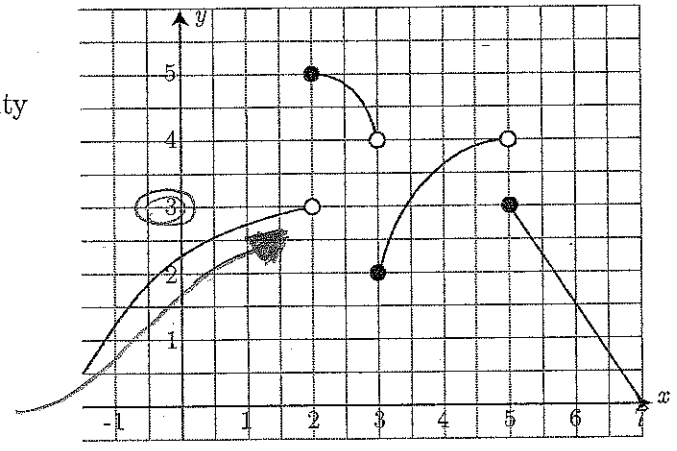
(a) The limit does not exist or approaches infinity

(b) 3

(c) 5

(d) 2

(e) 4




$\lim_{x\to 2^-} f(x)$ means approach 2 from the left

$$\lim_{x\to 2^-} f(x) = \boxed{3}$$

15. Consider the function $f(x) = \begin{cases} Ax^2 & \text{if } x < 3 \\ 13 - Ax & \text{if } x \geq 3 \end{cases}$

Find a value of $A$ so that the function is continuous at $x = 3$.

**Possibilities:**

(a) $\frac{3}{4}$

(b) $\frac{5}{6}$

(c) $\frac{11}{12}$

(d) 1

(e) $\frac{13}{12}$

To be continuous at $x=3$ we need

$$A(3)^2 = 13 - A(3)$$

$$9A = 13 - 3A$$

$$12A = 13$$

$$\boxed{A = \frac{13}{12}}$$

16. Find all values of $x$ where the derivative is not defined for $f(x) = |x^2 - 8x + 15|$.

**Possibilities:**

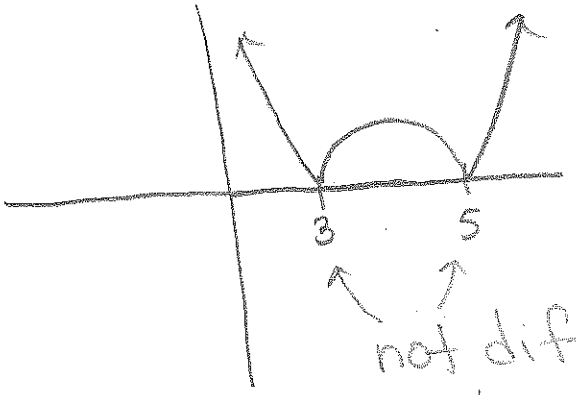
(a) x = -8 only

(b) x = 0 and x = 15

(c) x = 15 only

(d) x = -8 and x = 15

(e) x = 3 and x = 5




not differentiable at corners

$x^2 - 8x + 15$

$= (x-5)(x-3)$

which is zero at $\boxed{x=3 \text{ and } x=5}$

17. Find the equation of the tangent line to the graph of the function $f(x) = \dfrac{1}{x^2+1} + 4$ at $x = 3$. You may use $f'(x) = -\dfrac{2x}{(x^2+1)^2}$

**Possibilities:**

(a) $y = x^3 + 17$

(b) $y = -\dfrac{3}{50}x + \dfrac{107}{25}$

(c) $y = -\dfrac{3}{50}x + \dfrac{41}{10}$

(d) $y = \dfrac{41}{10}$

(e) $y = \dfrac{41}{10}x - \dfrac{309}{25}$

The equation of a line with slope $m$ passing through the point $(x_1, y_1)$ is

$y - y_1 = m(x - x_1)$

$f(3)$ $\quad f'(3)$ $\quad 3$

$f'(3) = \dfrac{-2(3)}{(3^2+1)^2} = \dfrac{-6}{10^2} = \dfrac{-6}{100} = \dfrac{-3}{50}$

$f(3) = \dfrac{1}{3^2+1} + 4 = \dfrac{1}{10} + 4 = \dfrac{41}{10}$

$y - \dfrac{41}{10} = \dfrac{-3}{50}(x-3) \Rightarrow y = \dfrac{-3}{50}x + \dfrac{9}{50} + \dfrac{41}{10}$

$\Rightarrow y = \dfrac{-3}{50}x + \dfrac{214}{50} \Rightarrow \boxed{y = \dfrac{-3}{50}x + \dfrac{107}{25}}$

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18. Consider the function $f(x) = 2x^2 + 3x + 5$. Its tangent line at $x = 3$ goes through the point $(6, y_1)$ where $y_1$ is:

**Possibilities:**

(a) 77

(b) 27

(c) 32

(d) 15

(e) $-13$

Find the equation of the tangent line first.

$y - y_1 = m(x - x_1)$

$f(3)$ $\quad f'(3)$ $\quad 3$

$f(3) = 2 \cdot 3^2 + 3 \cdot 3 + 5 = 32$

$f'(x) = 4x + 3 \Rightarrow f'(3) = 4(3) + 3 = 15$

$\Rightarrow y - 32 = 15(x-3) \Rightarrow y = 15x - 45 + 32 \Rightarrow y = 15x - 13$

plug in 6

$y_1 = 15(6) - 13 = \boxed{77}$

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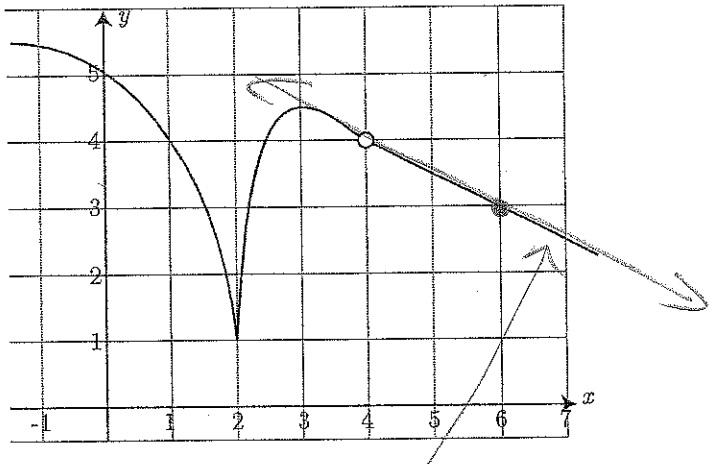
19. The graph of $y = f(x)$ is shown below. $f'(6)$ is approximately :




**Possibilities:**

(a) 2

(b) 3

(c) $-2$

(d) $\frac{1}{2}$

(e) $-\frac{1}{2}$

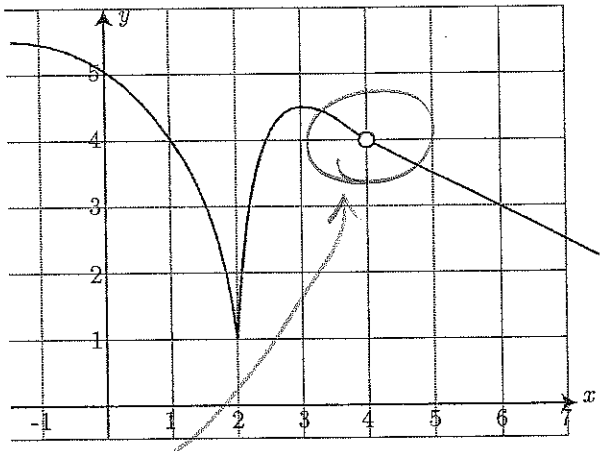
$f'(6)$ is slope of tangent line to the graph at $x=6$

Use points $(4,4)$ & $(6,3)$

slope $= \frac{4-3}{4-6} = -\frac{1}{2}$

Slope $= -\frac{1}{2}$

20. The graph of $y = f(x)$ is shown below. The function is continuous, except at $x =$




**Possibilities:**

(a) x = 2 only

(b) x = 2 and x = 4

(c) x = 4 only

(d) x = 2, x = 3, and x = 4

(e) x = 3 and x = 4

The function is continuous everywhere except at the hole.

$x = 4$