MA123 — Elem. Calculus Fall 2017
Exam 1 2017-09-21

Name: Solutions Sec.: _____

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and twenty multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name and section number on this page.

GOOD LUCK!

3.	a	b	**c**	d	e	12.	**a**	b	c	d	e
4.	a	**b**	c	d	e	13.	a	**b**	c	d	e
5.	a	b	c	d	**e**	14.	a	b	c	d	**e**
6.	**a**	b	c	d	e	15.	a	b	c	**d**	e
7.	a	b	c	**d**	e	16.	**a**	b	c	d	e
8.	a	**b**	c	d	e	17.	a	b	**c**	d	e
9.	a	b	c	**d**	e	18.	a	**b**	c	d	e
10.	a	b	c	d	**e**	19.	a	b	c	d	**e**
11.	a	b	**c**	d	e	20.	a	b	c	**d**	e

For grading use:

Multiple Choice	Short Answer
(number right) (5 points each)	(out of 10 points)

Total	
	(out of 100 points)

Solutions

Fall 2017 Exam 1 Short Answer Questions

*Write answers on this page. You must **show appropriate clear steps** to be sure you will get full credit.*

1. Evaluate the limit: $\lim_{x\to 5}\dfrac{x^2-4x-5}{x^2-6x+5}$

Plug in $x=5$: $\dfrac{(5)^2-4(5)-5}{(5)^2-6(5)+5}=\dfrac{0}{0}$

Thus we must simplify first

$$=\lim_{x\to 5}\frac{(x-5)(x+1)}{(x-5)(x-1)}$$

$$=\lim_{x\to 5}\frac{x+1}{x-1}=\frac{5+1}{5-1}=\frac{6}{4}=\frac{3}{2}$$

Final answer: $\dfrac{3}{2}$

2. Let $f(x)=x^2+3x+10$. Find the **equation** of the tangent line to $f(x)$ at $x=1$.

Need to find slope and a point

slope $=f'(1)$

$f'(x)=2x+3$

$f'(1)=2(1)+3=5$

point: $x=1 \Rightarrow y=f(1)=(1)^2+3(1)+10$

$=1+3+10=14$

Equation: $y-14=5(x-1)$

OR in $y=mx+b$ form as

$y-14=5x-5 \Rightarrow y=5x+9$

Equation of tangent line: $y-14=5(x-1)$

Name: Solutions

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

3. The expression

$$\frac{x^{24}\,(2x)^6}{x^8}$$

can be simplified to which of the following?

Possibilities:

(a) $2x^{22}$

(b) $64x^{14}$

(c) $64x^{22}$

(d) $2x^{14}$

(e) $2x^{18}$

$$= \frac{x^{24} \cdot 2^6 x^6}{x^8}$$

$$= \frac{64 x^{30}}{x^8}$$

$$= \boxed{64 x^{22}}$$

4. Find the domain of the function

$$f(x) = \sqrt{6 - x}.$$

Possibilities:

(a) $[0, \infty]$

(b) $(-\infty, 6]$

(c) $[6, \infty)$

(d) $(-\infty, 6)$

(e) $(6, \infty)$

$$6 - x \geq 0$$

$$6 \geq x$$

$$\Rightarrow \boxed{(-\infty, 6]}$$

5. If $h(t)$ represents the height of an object in feet above ground level at time t seconds and $h(t)$ is given by $h(t) = -16t^2 + 21t + 138$, find the time at which the speed of the object is zero.

Possibilities:

(a) 138 seconds

(b) (53/32) seconds

(c) (21/16) seconds

(d) (69/16) seconds

(e) (21/32) seconds

Speed is 0 when $h'(t) = 0$.

$h'(t) = -32t + 21$

$-32t + 21 = 0$

$21 = 32t$

$t = \frac{21}{32}$ sec

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6. If $f(x) = \sqrt{x+4}$ then choose the simplified form of $\frac{f(x+h)-f(x)}{h}$:

Possibilities:

(a) $\frac{1}{\sqrt{x+h+4}+\sqrt{x+4}}$

(b) $\frac{\frac{1}{2}}{\sqrt{x+h+4}}$

(c) 1

(d) $\frac{1}{2}\sqrt{x+h+4} - \frac{1}{2}\sqrt{x+4}$

(e) $\frac{h\sqrt{x+4}+\frac{1}{2}}{\sqrt{x+4}}$

$= \frac{\sqrt{x+h+4} - \sqrt{x+4}}{h} \left(\frac{\sqrt{x+h+4} + \sqrt{x+4}}{\sqrt{x+h+4} + \sqrt{x+4}} \right)$

$= \frac{x+h+4 - (x+4)}{h\left(\sqrt{x+h+4} + \sqrt{x+4}\right)}$

$= \frac{x+h+4-x-4}{h\left(\sqrt{x+h+4} + \sqrt{x+4}\right)}$

$= \frac{h}{h\left(\sqrt{x+h+4} + \sqrt{x+4}\right)}$

$= \frac{1}{\sqrt{x+h+4} + \sqrt{x+4}}$

7. The graph of $y = f(x)$ is shown below. Compute the average rate of change of $f(x)$ from $x = 1$ to $x = 3$.

Possibilities:

(a) $\frac{2}{5}$

(b) $\frac{1}{5}$

(c) $\frac{2}{3}$

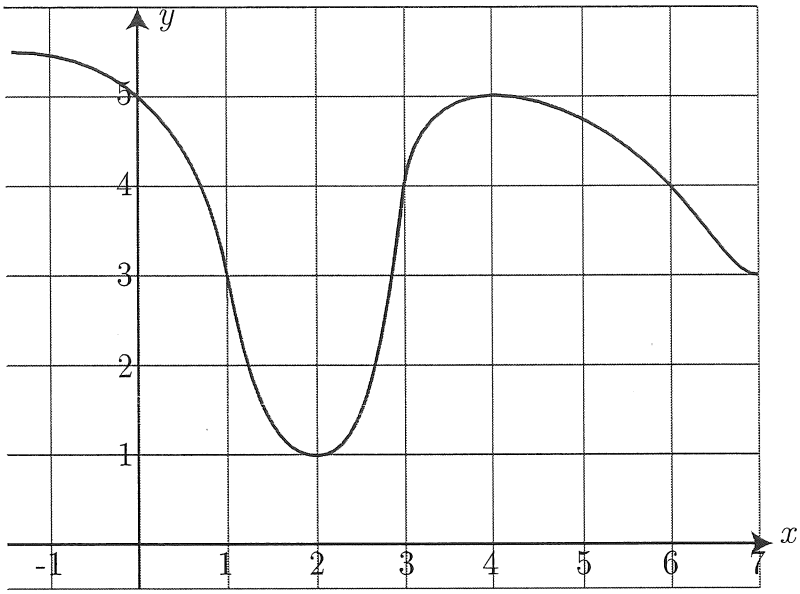
(d) $\frac{1}{2}$

(e) 2

Average rate of f change from $x=1$ to $x=3$ is $\frac{f(3)-f(1)}{3-1}$

$= \frac{4-3}{3-1}$

$= \boxed{\frac{1}{2}}$



8. Let $f(x) = x^3$. Find a value c between $x = 0$ and $x = 8$, so that the average rate of change of $f(x)$ from $x = 0$ to $x = 8$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$. You may use the fact that $f'(x) = 3x^2$.

Possibilities:

(a) 192

(b) $\frac{8}{\sqrt{3}}$

(c) 6

(d) $\frac{8}{\sqrt{5}}$

(e) $\frac{\sqrt{3}}{8}$

AROC of f from $x=0$ to $x=8$ = IROC of f at $x=c$

$\frac{f(8)-f(0)}{8-0} = f'(c)$

$\frac{(8)^3-(0)^3}{8} = 3c^2$

$(8)^2 = 3c^2$

$\frac{64}{3} = c^2$

$\Rightarrow c = \pm\sqrt{\frac{64}{3}}$

$= \pm\frac{8}{\sqrt{3}}$

c must be between 0 and 8, so it can't be negative.

$\Rightarrow \boxed{c = +\frac{8}{\sqrt{3}}}$

9. If $\lim_{x\to 3} f(x) = 11$ and $\lim_{x\to 3} g(x) = 17$, then what is the value of $\lim_{x\to 3} \dfrac{(x+5)(f(x)+1)}{g(x)}$?

Possibilities:

(a) $\dfrac{11}{17}$

(b) 0

(c) the limit is infinity or does not exist

(d) $\dfrac{(3+5)(11+1)}{17}$

(e) $\dfrac{(3)(11)}{17}$

$$= \frac{\lim_{x\to 3}(x+5) \cdot \lim_{x\to 3}(f(x)+1)}{\lim_{x\to 3} g(x)}$$

$$= \frac{(3+5)(11+1)}{17}$$

10. Find the limit

$$\lim_{x\to 36} \frac{x^2-16}{x-36}$$

Plug in $x=36$: $\dfrac{(36)^2-16}{0} = \dfrac{\text{Nonzero}}{0}$

This limit does not exist or it tends to ∞

Possibilities:

(a) 0

(b) $\dfrac{16}{36}$

(c) 40

(d) 1

(e) This limit either tends to infinity or this limit fails to exist

11. Find the limit

$$\lim_{t\to 1} \frac{36\sqrt{t}}{t}$$

Plug in $t=1$: $\frac{36\sqrt{1}}{1} = \frac{36(1)}{1} = \boxed{36}$

Possibilities:

(a) 18

(b) $\frac{18}{\sqrt{t}}$

(c) 36

(d) 0

(e) This limit either tends to infinity or this limit fails to exist

12. Find the limit

$$\lim_{n\to\infty} \frac{(4n+3)^2}{13n^5+4n^2+11} = \lim_{n\to\infty} \frac{16n^2+24n+9}{13n^5+4n^2+11}$$

$$= \lim_{n\to\infty} \frac{16n^2}{13n^5}$$

For infinite limits, we can ignore smaller power terms

$$= \lim_{n\to\infty} \frac{16}{13n^3}$$

$$= \frac{16}{\infty} = \boxed{0}$$

Possibilities:

(a) 0

(b) $\frac{4}{13}$

(c) The limit does not exist or approaches infinity

(d) $\frac{16}{13}$

(e) $\frac{16}{11}$

13. For the function

$$f(x) = \begin{cases} |4+8x| & \text{if } x < -2 \\ \sqrt{x^2+6} & \text{if } -2 \le x < 3 \\ 3x^2+x+5 & \text{if } 3 \le x \end{cases}$$

find $\lim_{x\to 5^+} f(x)$

use bottom piece.

$$\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+} 3x^2+x+5 = 3(5)^2+5+5$$
$$= 3(25)+10$$
$$= \boxed{85}$$

Possibilities:

(a) $\sqrt{31}$

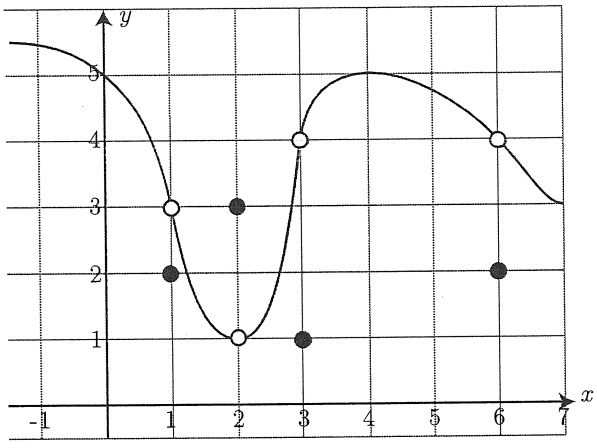
(b) 85

(c) $\sqrt{15}$

(d) 35

(e) 44

14. The graph of $y = f(x)$ is shown below. Compute $\lim_{x\to 1} f(x)$.



Possibilities:

(a) The limit does not exist or approaches infinity

(b) 0

(c) 2

(d) 1

(e) 3

$$\lim_{x\to 1} f(x) = \boxed{3}$$

15. Consider the function $f(x) = \begin{cases} x^2 - 4 & \text{if } x < 8 \\ 2x + B & \text{if } x \geq 8 \end{cases}$

Find a value of B so that the function is continuous at $x = 8$.

Possibilities:

(a) 41

(b) 42

(c) 43

(d) 44

(e) 45

Need $\lim_{x \to 8^-} f(x) = \lim_{x \to 8^+} f(x) = f(8)$

$\lim_{x \to 8^-} f(x) = \lim_{x \to 8^-} x^2 - 4 = (8)^2 - 4 = 60$

$\lim_{x \to 8^+} f(x) = \lim_{x \to 8^+} 2x + B = 2(8) + B = 16 + B$

$f(8) = 2(8) + B = 16 + B$

Set $16 + B = 60$

$\boxed{B = 44}$

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16. Find all values of x where the derivative is not defined for $f(x) = |x^2 - 14x + 45|$.

Possibilities:

(a) x = 5 and x = 9

(b) x = -14 and x = 45

(c) x = -14 only

(d) x = 45 only

(e) x = 0 and x = 45

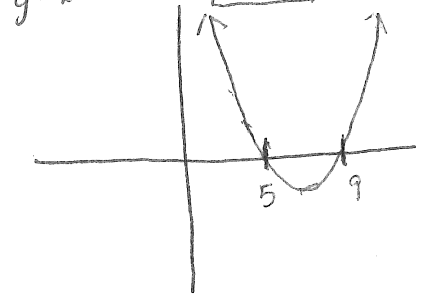
Draw graph of $y = x^2 - 14x + 45$

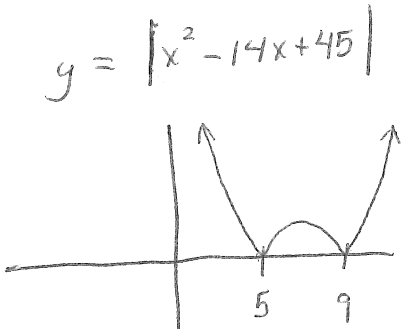
x-int's: set $y = 0$

$0 = x^2 - 14x + 45$

$0 = (x - 9)(x - 5)$

$x = 9, 5$





The derivative is not defined at $\boxed{x = 5, 9}$ since the graph has sharp corners there

17. Suppose that for a function $f(x)$, we know that

$$\frac{f(x+h)-f(x)}{h} = \frac{-2xh-h^2-8h}{h(x+8)^2(x+h+8)^2}.$$

Find the slope of the tangent line at $x = 7$.

Possibilities:

(a) $\frac{-14}{15^4}$

(b) $\frac{-14}{15^2}$

(c) $\frac{-22}{15^4}$

(d) 0

(e) The slope does not exist.

Slope of tangent line at $x=7$ $= \lim_{h\to 0} \frac{f(7+h)-f(7)}{h}$

$= \lim_{h\to 0} \frac{-2(7)h - h^2 - 8h}{h(15)^2(15+h)^2}$

$= \lim_{h\to 0} \frac{-h^2-22h}{h(15)^2(15+h)^2}$

$= \lim_{h\to 0} \frac{-h(h+22)}{h(15)^2(15+h)^2}$

$= \lim_{h\to 0} \frac{-(h+22)}{(15)^2(15+h)^2} = \frac{-22}{(15)^2(15+0)^2} = \boxed{\frac{-22}{15^4}}$

18. Consider the function $f(x) = x^2 + 4x + 7$. Its tangent line at $x = 3$ goes through the point $(6, y_1)$ where y_1 is:

Get equation of tangent line first.

Possibilities:

(a) 10

(b) 58

(c) 16

(d) -2

(e) 28

slope $= f'(3)$

$f'(x) = 2x+4$

$f'(3) = 2(3)+4 = 6+4 = 10$

point: $x=3 \Rightarrow y = (3)^2+4(3)+7$

$= 9+12+7 = 28$

Equation: $y-28 = 10(x-3)$

$y-28 = 10x-30$

$y = 10x-2$

To get y_1, we plug in $x=6$ into the eqn. of the tangent line.

$y = 10(6)-2$

$= \boxed{58}$

19. Determine the value of $f'(1)$ from the graph of $f(x)$ given here:

Possibilities:

(a) $f'(1) = 0$

(b) $f'(1) = -3$

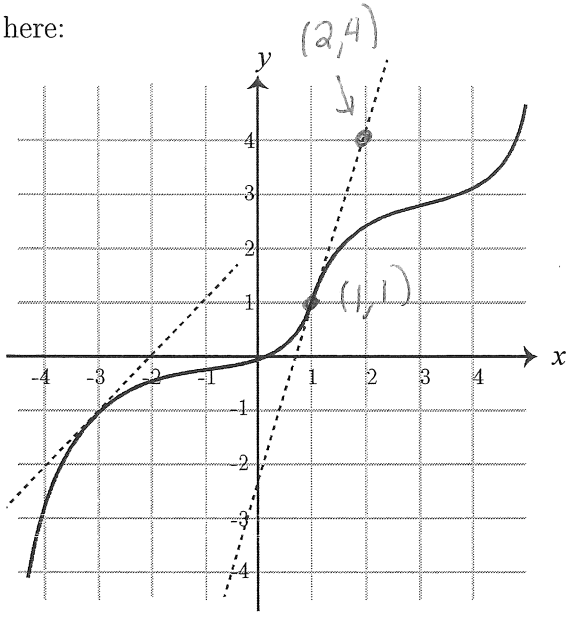
(c) $f'(1) = 1$

(d) $f'(1) = -1$

(e) $f'(1) = 3$

$f'(1)$ = slope of tangent line at $x=1$

$= \frac{4-1}{2-1} = \frac{3}{1} = \boxed{3}$



20. The graph of $y = f(x)$ is shown below. The function is **continuous**, except at $x =$

Possibilities:

(a) x=1, x=3, and x=4

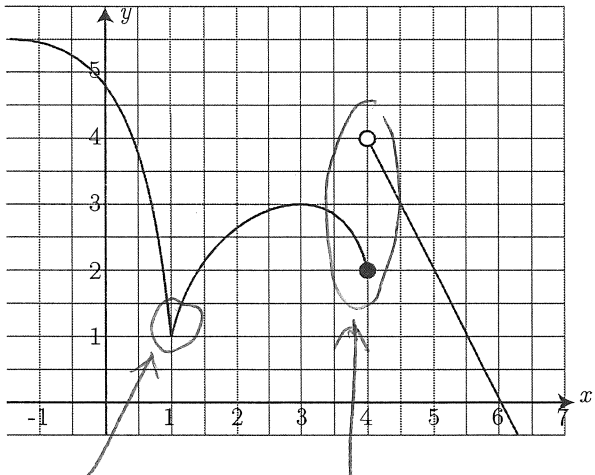
(b) x=1 and x=4

(c) x=1 only

(d) x=4 only

(e) x=1, x=3, x=4, and x=6

Not continuous at $x=4$



Not differentiable at $x=1$

Not continuous