MA123 — Elem. Calculus Fall 2018
Exam 1 2018-09-20

Name: SOLUTIONS Sec.: _____

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name and section number on this page.

GOOD LUCK!

3. (a) (b) (c) (d) (e)
4. (a) (b) (c) (d) (e)
5. (a) (b) (c) (d) (e)
6. (a) (b) (c) (d) (e)
7. (a) (b) (c) (d) (e)
8. (a) (b) (c) (d) (e)
9. (a) (b) (c) (d) (e)
10. (a) (b) (c) (d) (e)
11. (a) (b) (c) (d) (e)
12. (a) (b) (c) (d) (e)
13. (a) (b) (c) (d) (e)
14. (a) (b) (c) (d) (e)
15. (a) (b) (c) (d) (e)
16. (a) (b) (c) (d) (e)
17. (a) (b) (c) (d) (e)
18. (a) (b) (c) (d) (e)
19. (a) (b) (c) (d) (e)
20. (a) (b) (c) (d) (e)

For grading use:

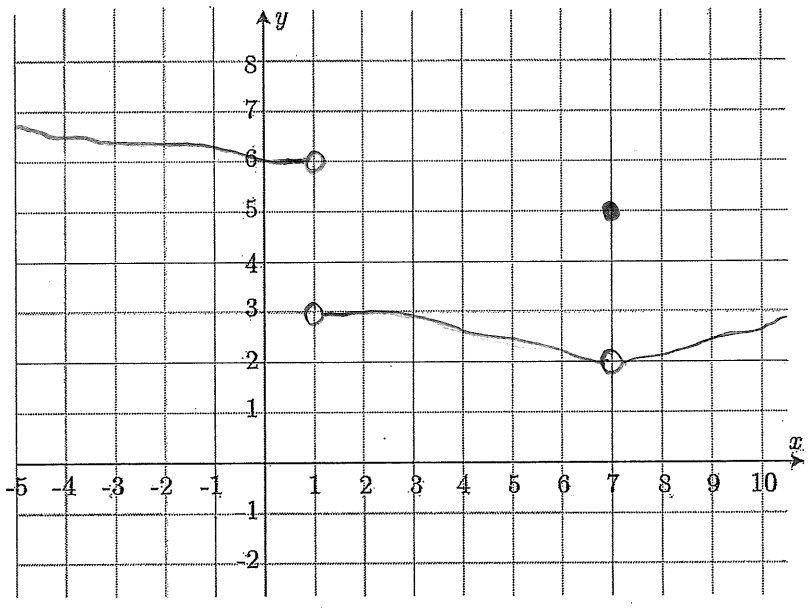
Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(out of 100 points)

Fall 2018 Exam 1 Short Answer Questions

Write answers on this page. Your work must be clear and legible to be sure you will get full credit.

1. Sketch the graph of a single function $y=f(x)$ which satisfies the following properties: $\lim_{x\to 1^-} f(x)=6$, $\lim_{x\to 1^+} f(x)=3$, $\lim_{x\to 7} f(x)=2$, $f(7)=5$, and $f(x)$ is continuous for all x except $x=1$ and $x=7$.



2. Let $f(x)=5x^2+8$. Find the **equation** of the tangent line to $f(x)$ at $x=3$. You do not need to simplify your answer.

$f(3)=5(3)^2+8 = 5\cdot 9+8 = 45+8 = 53$

$f'(x)=10x$

$f'(3)=10(3)=30$

$y=mx+b$ at point $(3,53)$

$53=30(3)+b$

$-90 \quad -90$

$-37=b$

$y=30x-37$

Equation of tangent line: $y=30x-37$

Name: _____

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

3. Simplify the expression

$$\frac{60x^{18} - 90x^{15} - 50x^7}{-10x^5}.$$

Possibilities:

(a) $-6x^{13} + 9x^{10} + 5x^2$

(b) $60x^{18} - 90x^{15} - 50x^7 + 10x^5$

(c) $-6x^{13} + 90x^{15} + 50x^7$

(d) $-60x^{18} + 90x^{15} + 5x^2$

(e) $-6x^{13} - 9x^{10} - 5x^2$

$= \frac{60x^{18}}{-10x^5} + \frac{-90x^{15}}{-10x^5} + \frac{-50x^7}{-10x^5}$

$= -6x^{13} + 9x^{10} + 5x^2$

4. Find the domain of the function

$$f(x) = \sqrt{7 - x}.$$

Possibilities:

(a) $[7, \infty)$

(b) $(-\infty, 7]$

(c) $(7, \infty)$

(d) $[0, \infty]$

(e) $(-\infty, 7)$

$7 - x \geq 0$

$7 \geq x$

$(-\infty, 7]$

5. Let $f(x) = x^2 - 62x + 9$. What is the value of x for which the tangent line to the graph of $y = f(x)$ is parallel to the $x-$axis?

Possibilities:

(a) 32

(b) 9

(c) -53

(d) -62

(e) 31

To find the value of x, find the derivative of f(x), set that equal to zero, and solve for x.

$f'(x) = 2x - 62$

$f'(x) = 0 = 2x - 62$

$62 = 2x$

$31 = x$

6. If $f(x) = \sqrt{x+4}$ then choose the simplified form of $\frac{f(x+h)-f(x)}{h}$:

Possibilities:

(a) 1

(b) $\dfrac{h\sqrt{x+4}+\frac{1}{2}}{\sqrt{x+4}}$

(c) $\dfrac{1}{\sqrt{x+h+4}+\sqrt{x+4}}$

(d) $\dfrac{\frac{1}{2}}{\sqrt{x+h+4}}$

(e) $\frac{1}{2}\sqrt{x+h+4} - \frac{1}{2}\sqrt{x+4}$

$f(x+h) = \sqrt{x+h+4}$, $f(x) = \sqrt{x+4}$

Plug in!

$$\frac{\sqrt{x+h+4} - \sqrt{x+4}}{h}$$

Simplify:

$$\frac{\sqrt{x+h+4} - \sqrt{x+4}}{h} \cdot \frac{\sqrt{x+h+4} + \sqrt{x+4}}{\sqrt{x+h+4} + \sqrt{x+4}}$$

$$= \frac{x+h+4 - (x+4)}{h(\sqrt{x+h+4} + \sqrt{x+4})}$$

$$= \frac{h}{h(\sqrt{x+h+4} + \sqrt{x+4})} = \frac{1}{\sqrt{x+h+4} + \sqrt{x+4}}$$

7. The graph of $y = f(x)$ is shown below. Compute the average rate of change of $f(x)$ from $x = -1$ to $x = 2$.

Possibilities:

(a) $\frac{1}{4}$

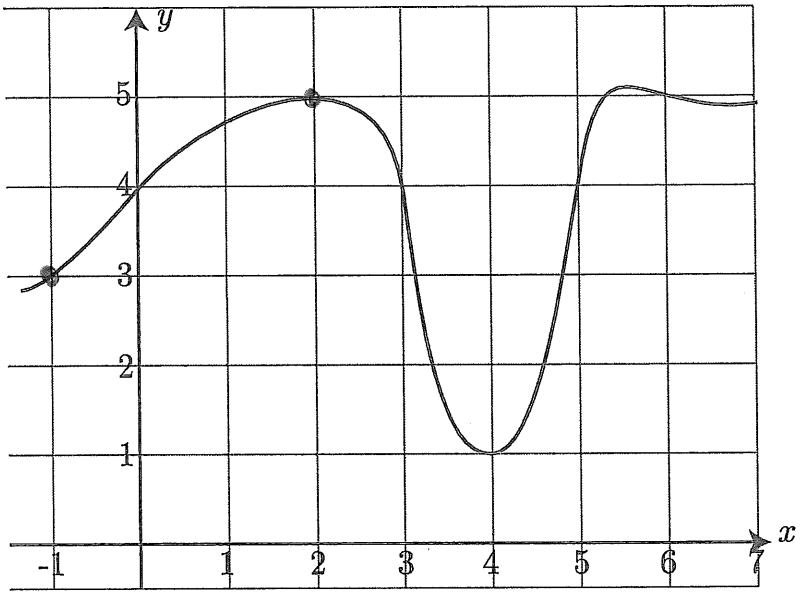
(b) $-\frac{2}{5}$

(c) $\frac{1}{6}$

(d) $\frac{2}{3}$

(e) $\frac{2}{7}$

$$\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{5-3}{2-(-1)} = \frac{2}{3}$$



8. Let $f(x) = x^3$. Find a value c between $x = 0$ and $x = 10$, so that the average rate of change of $f(x)$ from $x = 0$ to $x = 10$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$. You may use the fact that $f'(x) = 3x^2$.

Possibilities:

(a) $\frac{10}{\sqrt{3}}$

(b) 5

(c) 300

(d) $\frac{10}{\sqrt{5}}$

(e) $\frac{\sqrt{3}}{10}$

$$\text{AROC} = \frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{10^3-0^3}{10-0} = 100$$

$$\text{IROC} \Rightarrow f'(c) = 3c^2 = \frac{100}{3}$$

$$c^2 = \frac{100}{3}$$

$$c = \sqrt{\frac{100}{3}} = \frac{10}{\sqrt{3}}$$

9. If $\lim_{x\to 7} f(x) = 13$ and $\lim_{x\to 7} g(x) = 11$, then what is the value of $\lim_{x\to 7} \dfrac{17f(x)+2}{x+g(x)}$?

Possibilities:

(a) $\dfrac{13}{11}$

(b) 0

(c) $\dfrac{(17)(13)(7)+2}{7+(11)(7)}$

(d) $\dfrac{(17)13+2}{7+11}$

(e) the limit is infinity or does not exist

$$\frac{\lim_{x\to 7}(17f(x)+2)}{\lim_{x\to 7}(x+g(x))} = \boxed{\frac{17\cdot 13+2}{7+11}}$$

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10. Compute $\lim_{t\to 1} \dfrac{t^2-1}{t^2+5t-6}$

Possibilities:

(a) 0

(b) The limit does not exist.

(c) 1

(d) $\frac{9}{7}$

(e) $\frac{2}{7}$

$\lim_{t\to 1} \dfrac{t^2-1}{t^2+5t-6}$ ← testing $t=1$ gives $\frac{0}{0}$, so DO MORE WORK:

$= \lim_{t\to 1} \dfrac{(t-1)(t+1)}{(t+6)(t-1)}$

$= \lim_{t\to 1} \dfrac{t+1}{t+6}$

$= \dfrac{1+1}{1+6}$

$= \boxed{\frac{2}{7}}$

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11. Find the one-sided limit

$$\lim_{t\to 0^-} \frac{|36t|}{t}$$

Possibilities:

(a) This limit either tends to infinity or this limit fails to exist

(b) 36

(c) $\frac{18}{\sqrt{t}}$

(d) 0

(e) -36

Break into piecewise function:

$$|36t| = \begin{cases} -36t, & t<0 \\ 36t, & t \geq 0 \end{cases}$$

$\lim_{t\to 0^-} \frac{-36t}{t}$ ← since approaching 0 from the left side

$= -36$

t's cancel so you don't need to apply limit to a constant!

12. Find the limit

$$\lim_{x\to\infty} \frac{11x+17x^3+6}{19+x^2+13x^3} = \lim_{x\to\infty} \frac{17x^3}{13x^3} = \frac{17}{13}$$

Possibilities:

(a) $\frac{17}{13}$

(b) $\frac{34}{33}$

(c) 6

(d) The limit does not exist or approaches infinity

(e) $\frac{11}{19}$

$\lim_{x\to\infty}$ with same highest degree in numerator and denominator means that the limit is the leading coefficient in the numerator divided by the leading coefficient in the denominator!

$\frac{17}{13}$

13. Given the function $f(x) = \begin{cases} x & \text{if } x \leq 0 \\ 15x + 6 & \text{if } x > 0 \end{cases}$

evaluate the limit as x tends to zero from the right,

$$\lim_{x \to 0^+} f(x)$$

Possibilities:

(a) 15

(b) This limit does not exist

(c) 21

(d) 6

(e) 0

$x \to 0^+$ means to look at second expression, where the domain is $x > 0$.

$\lim_{x \to 0^+} 15x + 6$

$= 15 \cdot 0 + 6$

$= 6$

14. If $f(x) = x^2 + 4x + 7$ then choose the simplified form of $\frac{f(x+h)-f(x)}{h}$:

Possibilities:

(a) $2x + 2h + 4$

(b) $2x + 4 + h$

(c) $-2x - 4 - h$

(d) 6

(e) $\dfrac{h^2 + (2x+4)h + 2x^2 + 8x + 14}{2x + h}$

$f(x+h) = (x+h)^2 + 4(x+h) + 7$

$= x^2 + 2xh + h^2 + 4x + 4h + 7$

$f(x) = x^2 + 4x + 7$

Plug in!

$$\frac{x^2 + 2xh + h^2 + 4x + 4h + 7 - (x^2 + 4x + 7)}{h}$$

$$= \frac{2xh + h^2 + 4h}{h} = \frac{h(2x + h + 4)}{h}$$

$= 2x + h + 4$

15. Consider the function $f(x) = \begin{cases} Ax^3 & \text{if } x < 2 \\ 6 - Ax & \text{if } x \geq 2 \end{cases}$

Find a value of A so that the function is continuous at $x = 2$.

Possibilities:

(a) $\frac{1}{5}$

(b) $\frac{2}{5}$

(c) $\frac{3}{5}$

(d) $\frac{4}{5}$

(e) 1

$Ax^3 = 6 - Ax$ at $x = 2$

$A(2)^3 = 6 - A(2)$

$A \cdot 8 = 6 - 2A$

$+2A \quad +2A$

$\frac{10A}{10} = \frac{6}{10}$

$A = 6/10$

$A = 3/5$

16. The graph of $y = f(x)$ is shown below. $f'(6)$ is approximately :

Possibilities:

(a) $-\frac{1}{2}$

(b) $\frac{1}{2}$

(c) -2

(d) 3

(e) 2

① $x = 6$ on $f(x)$ is decreasing so $f'(6)$ will be negative.

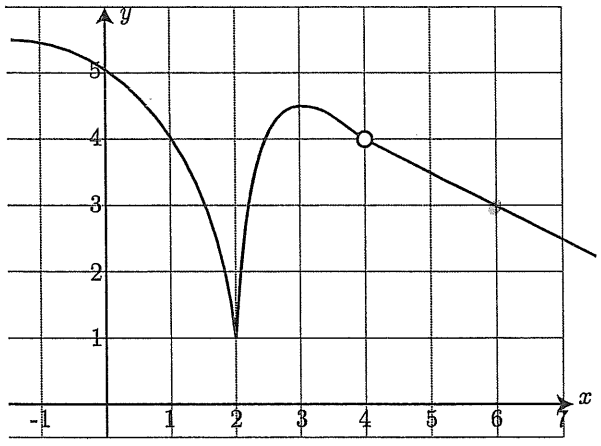
② On same tangent line at $x = 4$ so find slope!

using points $(6, 3)$ and $(4, 4)$

$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \frac{3 - 4}{6 - 4}$

$= -1/2$



17. For the function $f(x) = (x+8)^2$, find the equation of the tangent line to the graph of f at $x = 3$.

Possibilities:

(a) $y = 22x + 121$

(b) $y = 22x + 55$

(c) $y = 6x + 103$

(d) $y = 6x + 121$

(e) $y = x + 8$

$f(3) = (3+8)^2$

$= 11^2$

$= 121$ pt on $f(x)$ is $(3, 121)$

$f(x) = (x+8)^2 = x^2 + 16x + 64$

$f'(x) = 2x + 16$

$m = f'(3) = 2(3) + 16$

$= 22$

$y = mx + b$

$121 = 22(3) + b$

$b = 121 - 66 = 55$

So tangent line is $y = mx + b$

$\Rightarrow$ $y = 22x + 55$

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18. Consider the function $f(x) = x^2 + 4x + 7$. Its tangent line at $x = 3$ goes through the point $(9, y_1)$ where y_1 is:

Possibilities:

(a) 28

(b) -2

(c) 10

(d) 88

(e) 22

$f(3) = 3^2 + 4(3) + 7$

$= 9 + 12 + 7$

$= 28$ pt on $f(x) = (3, 28)$

$f'(x) = 2x + 4$

$m = f'(3) = 2(3) + 4$

$= 6 + 4$

$= 10$

$y = mx + b$

$28 = 10(3) + b$

$b = 28 - 30 = -2$

So tangent line is

$y = 10x - 2$

and contains pt $(9, y_1)$

$y_1 = 10(9) - 2$

$y_1 = 90 - 2$

$y_1 = 88$

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19. The graph of $y = f(x)$ is shown below. The function is **continuous**, except at $x =$

Possibilities:

(a) x=1, x=4, and x=5

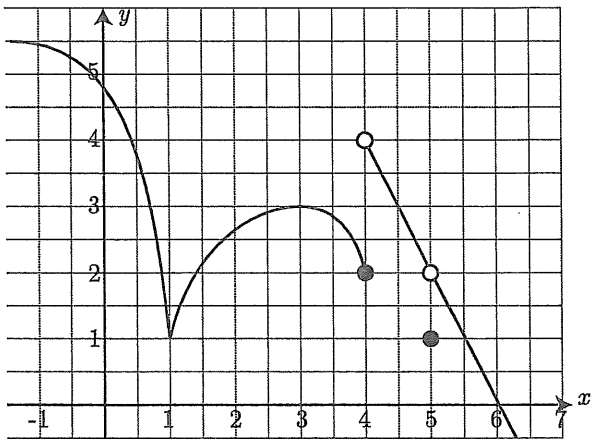
(b) x=1, x=3, x=4, and x=5

(c) x=4 and x=5

(d) x=1 and x=3

(e) x=4 only

Not cont'ts at holes!



20. The graph of $y = f(x)$ is shown below. The function is **differentiable**, except at $x =$

Possibilities:

(a) x=1, x=4, and x=5

(b) x=4 and x=5

(c) x=1, x=3, x=4, and x=5

(d) x=1 and x=3

(e) x=4 only

Not diff'ble at holes and sharp pts.