MA123 — Elem. Calculus Spring 2017
Exam 1 2017-02-09

Name: SOLUTIONS **Sec.:** _____

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name on this page, and at the top of page three.

GOOD LUCK!

3. (a) (b) **(c)** (d) (e)
4. (a) **(b)** (c) (d) (e)
5. (a) (b) (c) **(d)** (e)
6. **(a)** (b) (c) (d) (e)
7. (a) (b) **(c)** (d) (e)
8. (a) (b) (c) (d) **(e)**
9. (a) (b) (c) **(d)** (e)
10. (a) **(b)** (c) (d) (e)
11. (a) (b) **(c)** (d) (e)
12. **(a)** (b) (c) (d) (e)
13. (a) (b) (c) **(d)** (e)
14. (a) (b) (c) (d) **(e)**
15. (a) (b) (c) **(d)** (e)
16. **(a)** (b) (c) (d) (e)
17. (a) (b) **(c)** (d) (e)
18. (a) (b) (c) **(d)** (e)
19. (a) **(b)** (c) (d) (e)
20. (a) (b) (c) **(d)** (e)

For grading use:

Multiple Choice	Short Answer
(number right) (5 points each)	(out of 10 points)

Total	
	(total 100 points)

Name: **Last 4 digits of Student ID:**

Spring 2017 Exam 1 Short Answer Questions

Write answers on this page. You must show appropriate legible steps to be sure you will get full credit.

1. Let $f(x)=5x^2-7x+8$. Find a value of x such that the slope of the tangent line to the graph of $f(x)$ equals 23 at that x value.

Slope of tangent line $= f'(x) = 10x-7$ (derivative of ax^2+bx+c is $2ax+b$)

Set $f'(x)=23$ and solve:

$23 = 10x-7$

$30 = 10x$

$\boxed{3=x}$

2. Let $f(x)=4x^2+10$. Find the average rate of change of $f(x)$ with respect to x as x changes from 3 to $3+h$. **Simplify** your answer, and circle your final answer.

$$AROC = \frac{f(b)-f(a)}{b-a} \quad \text{(as } x \text{ changes from } a \text{ to } b\text{)}$$

$$= \frac{f(3+h)-f(3)}{3+h-3}$$

$$= \frac{4(3+h)^2+10-\left(4(3)^2+10\right)}{h}$$

$$= \frac{4(9+6h+h^2)+10-4(9)-10}{h}$$

$$= \frac{36+24h+4h^2-36}{h}$$

$$= \frac{h(24+4h)}{h} = 24+4h$$

Name: _____

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

3. Solve the equation $4x^2 + 106xy + 2y = 9$ for y in terms of x

Possibilities:

(a) $y = \dfrac{-106 \pm \sqrt{11204}}{8}$

(b) $y = \dfrac{9 - 4x^2 - 106x}{2}$

(c) $y = \dfrac{9 - 4x^2}{106x + 2}$

(d) $y = \dfrac{4x^2 - 9}{106x + 2}$

(e) $y = \dfrac{106x + 2}{4x^2 - 9}$

Get all the y's on one side:

$106xy + 2y = 9 - 4x^2$

$y(106x + 2) = 9 - 4x^2$

$y = \dfrac{9 - 4x^2}{106x + 2}$

4. Evaluate $f(3)$ when $f(x)$ is given by the piecewise definition

$$f(x) = \begin{cases} x^2 - 7 & \text{if } x \leq 2 \\ 4x - 1 & \text{if } 2 < x < 3 \\ x^2 - 8x & \text{if } 3 \leq x \end{cases}$$

← Plug into this piece

Possibilities:

(a) 2

(b) -15

(c) 7

(d) 11

(e) DNE

$f(3) = (3)^2 - 8(3)$

$= 9 - 24$

$= -15$

5. If $h(t)$ represents the height of an object in feet above ground level at time t seconds and $h(t)$ is given by $h(t) = -16t^2 + 13t + 110$, find the height of the object at the time when the speed of the object is zero.

Possibilities:

(a) (13/16) feet

(b) 110 feet

(c) (7273/64) feet

(d) (7209/64) feet

(e) (13/32) feet

The speed is zero when $h'(t) = 0$.

$h'(t) = -32t + 13 = 0$

$13 = 32t$

$\frac{13}{32} = t$

To find the height of the object at $t = \frac{13}{32}$, plug $t = \frac{13}{32}$ into $h(t)$.

$h\left(\frac{13}{32}\right) = -16\left(\frac{13}{32}\right)^2 + 13\left(\frac{13}{32}\right) + 110 = \frac{7209}{64}$

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6. If $f(x) = \sqrt{x+2}$ then choose the simplified form of $\frac{f(x+h)-f(x)}{h}$:

Possibilities:

(a) $\frac{1}{\sqrt{x+h+2}+\sqrt{x+2}}$

(b) $\frac{1}{2}\sqrt{x+h+2} - \frac{1}{2}\sqrt{x+2}$

(c) $\frac{h\sqrt{x+2}+\frac{1}{2}}{\sqrt{x+2}}$

(d) $\frac{\frac{1}{2}}{\sqrt{x+h+2}}$

(e) 1

$\frac{f(x+h)-f(x)}{h}$

$= \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h}$ Multiply by the conjugate

$= \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h} \left(\frac{\sqrt{x+h+2}+\sqrt{x+2}}{\sqrt{x+h+2}+\sqrt{x+2}}\right)$

$= \frac{x+h+2-(x+2)}{h\left(\sqrt{x+h+2}+\sqrt{x+2}\right)}$

$= \frac{h}{h\left(\sqrt{x+h+2}+\sqrt{x+2}\right)} = \frac{1}{\sqrt{x+h+2}+\sqrt{x+2}}$

---

7. For the function $f(x) = 5x^2 + 3x + 2$, find the equation of the tangent line to the graph of f at $x = 5$.

Possibilities:

(a) $y = 142$

(b) $y = 53x + 142$

(c) $y = 53x - 123$

(d) $y = 142x - 657$

(e) $y = x^3 + 17$

Need slope and point:

Slope $= f'(5)$

$f'(x) = 10x + 3$

$f'(5) = 50 + 3 = 53$

Point $= (5, y)$

Get y by plugging $x = 5$ into $f(x)$.

$y = 5(5)^2 + 3(5) + 2$

$= 5(25) + 15 + 2$

$= 125 + 15 + 2 = 142$

→ write equation in point-slope form

$y - y_1 = m(x - x_1)$

$y - 142 = 53(x - 5)$

$y = 53x - 265 + 142$

$y = 53x - 123$

---

8. Let $f(x) = 5x^2 + 3x + 2$. Find a value c between $x = 3$ and $x = 7$, so that the average rate of change of $f(x)$ from $x = 3$ to $x = 7$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$.

Possibilities:

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

$AROC = \frac{f(7) - f(3)}{7 - 3} = \frac{5(49) + 3(7) + 2 - (5(9) + 3(3) + 2)}{4}$

$= \frac{245 + 21 + 2 - 45 - 9 - 2}{4} = \frac{212}{4} = 53$

Instantaneous rate of change at $x = c$ is $f'(c)$

$f'(x) = 10x + 3$

$f'(c) = 10c + 3$

Set $f'(c) = 53$ and solve

$10c + 3 = 53 \Rightarrow 10c = 50 \Rightarrow \boxed{c = 5}$

9. If $\lim_{x\to 11} f(x) = 7$ and $\lim_{x\to 11} g(x) = 5$, then what is the value of $\lim_{x\to 11} \dfrac{(x+17)(f(x)+1)}{g(x)}$?

Possibilities:

(a) 0

(b) the limit is infinity or does not exist

(c) $\frac{7}{5}$

(d) $\dfrac{(11+17)(7+1)}{5}$

(e) $\dfrac{(11)(7)}{5}$

$$= \frac{\left(\lim_{x\to 11} x+17\right)\left(\lim_{x\to 11} f(x)+1\right)}{\lim_{x\to 11} g(x)}$$

$$= \frac{(11+17)(7+1)}{5}$$

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10. Compute $\lim_{t\to 8} \dfrac{t^2-t-56}{t^2-3t-40}$

Possibilities:

(a) $\frac{14}{13}$

(b) $\frac{15}{13}$

(c) $\frac{16}{13}$

(d) $\frac{17}{13}$

(e) The limit does not exist.

Plug in $t=8$:

$$\frac{(8)^2-8-56}{(8)^2-3(8)-40} = \frac{64-8-56}{64-24-40} = \frac{0}{0}$$

Try factoring/simplifying

$$\lim_{t\to 8} \frac{(t-8)(t+7)}{(t-8)(t+5)}$$

$$= \lim_{t\to 8} \frac{t+7}{t+5}$$

Plug in $t=8$: $\dfrac{8+7}{8+5} = \dfrac{15}{13}$

---

11. Find the limit

$$\lim_{t\to 0^+} \frac{50\sqrt{t}}{t}$$

$= \lim_{t\to 0^+} \frac{50}{\sqrt{t}}$ (cancel $\sqrt{t}$)

$= \lim_{t\to 0^+} \frac{50\sqrt{t}}{\sqrt{t}\cdot\sqrt{t}}$ =

Plug in $t=0$:

$\frac{50}{0}$

This limit does not exist

Possibilities:

(a) 25

(b) $\frac{25}{\sqrt{t}}$

(c) This limit either tends to infinity or this limit fails to exist

(d) 0

(e) 50

---

12. Find the limit

$$\lim_{x\to\infty} \frac{7x+11x^3+6}{13+23x^3+x^2}$$

We consider the highest power terms

$= \lim_{x\to\infty} \frac{11x^3}{23x^3}$ Cancel the x^3

$= \lim_{x\to\infty} \frac{11}{23}$

$= \frac{11}{23}$

Possibilities:

(a) $\frac{11}{23}$

(b) The limit does not exist or approaches infinity

(c) $\frac{7}{13}$

(d) $\frac{24}{37}$

(e) 6

13. For the function

$$f(x) = \begin{cases} |2+3x| & \text{if } x < -1 \\ \sqrt{x^2+3} & \text{if } -1 \le x < 2 \\ 8x^2+9x+4 & \text{if } 2 \le x \end{cases}$$

find $\lim_{x\to 6^+} f(x)$

Possibilities:

(a) 54

(b) $\sqrt{39}$

(c) $\sqrt{7}$

(d) 346

(e) 20

We use this piece because six is larger than 2

$\lim_{x\to 6^+} f(x) = \lim_{x\to 6^+} 8x^2+9x+4$

Plug in $x=6$!

$8(6)^2+9(6)+4$

$= 346$

14. The graph of $y = f(x)$ is shown below. Compute $\lim_{x\to 2^-} f(x)$.

Possibilities:

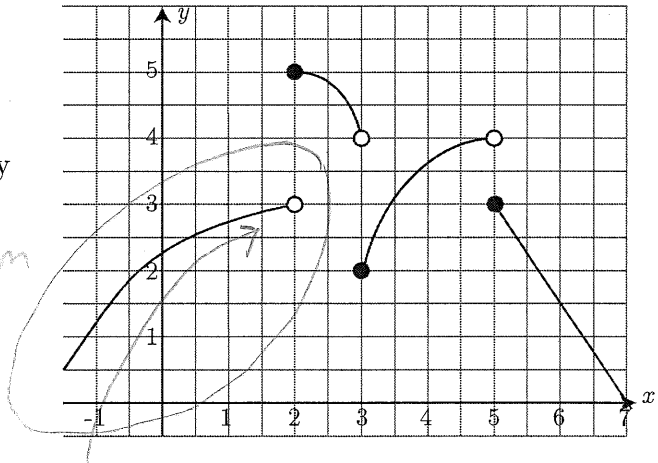
(a) 5

(b) The limit does not exist or approaches infinity

(c) 4

(d) 2

(e) 3



We approach $x=2$ from the left.

$\lim_{x\to a^-} f(x)$ is what the outputs (y-values) approach,

which is 3

15. Consider the function $f(x) = \begin{cases} Ax^2 & \text{if } x < 3 \\ 11 - Ax & \text{if } x \geq 3 \end{cases}$

Find a value of A so that the function is continuous at $x = 3$.

Possibilities:

(a) $\frac{2}{3}$

(b) $\frac{3}{4}$

(c) $\frac{5}{6}$

(d) $\frac{11}{12}$

(e) 1

Set $A(3)^2 = 11 - A(3)$

$A(9) = 11 - 3A$

$9A = 11 - 3A$

$12A = 11$

$A = \frac{11}{12}$

why?

We require $\lim_{x \to 3} f(x)$ to exist,

so $\lim_{x \to 3^-} f(x)$ must equal $\lim_{x \to 3^+} f(x)$.

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} Ax^2$

$= A \cdot 3^2 = 9A$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (11 - Ax)$

$= 11 - A(3) = 11 - 3A.$

16. Find all values of x where the derivative is not defined for $f(x) = |x^2 - 12x + 32|$.

Possibilities:

(a) x = 4 and x = 8

(b) x = -12 and x = 32

(c) x = 0 and x = 32

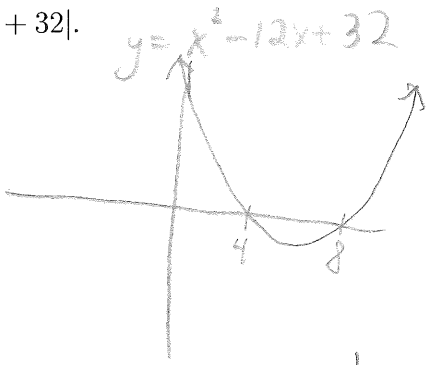
(d) x = 32 only

(e) x = -12 only

$y = x^2 - 12x + 32$ is a parabola

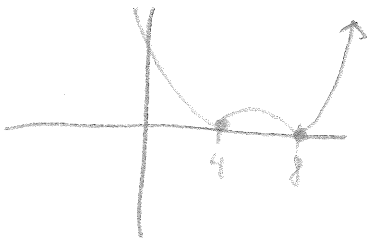
$= (x-4)(x-8)$

The x-intercepts are 4 and 8.



The derivative is not defined at the corners, $x = 4$ and $x = 8$.

$y = |x^2 - 12x + 32|$



17. Suppose that for a function $f(x)$, we know that

$$\frac{f(x+h)-f(x)}{h} = \frac{-2xh-h^2-7h}{h(x+7)^2(x+h+7)^2}.$$

Find the slope of the tangent line at $x = 6$.

Possibilities:

(a) The slope does not exist.

(b) $\frac{-12}{13^2}$

(c) $\frac{-19}{13^4}$

(d) $\frac{-12}{13^4}$

(e) 0

$f'(6)$ = slope of the tangent line at $x=6$ $= \lim_{h\to 0} \frac{f(6+h)-f(6)}{h}$

$= \lim_{h\to 0} \frac{-2(6)h-h^2-7h}{h(6+7)^2(6+h+7)^2}$

$= \lim_{h\to 0} \frac{-19h-h^2}{h(13)^2(13+h)^2}$

$= \lim_{h\to 0} \frac{h(-19-h)}{h(13)^2(13+h)^2}$

$= \lim_{h\to 0} \frac{-19-h}{(13)^2(13+h)^2}$

Plug in $h=0$:

$\frac{-19}{(13)^2(13)^2} = \frac{-19}{13^4}$

18. Let $f(x) = x^2 - 42x + 8$. What is the value of x for which the tangent line to the graph of $y = f(x)$ is parallel to the $x-$axis?

Possibilities:

(a) -42

(b) 8

(c) -34

(d) 21

(e) 22

Tangent line is parallel to x-axis when $f'(x)=0$:

$f'(x) = 2x - 42$

Set $f'(x)=0$

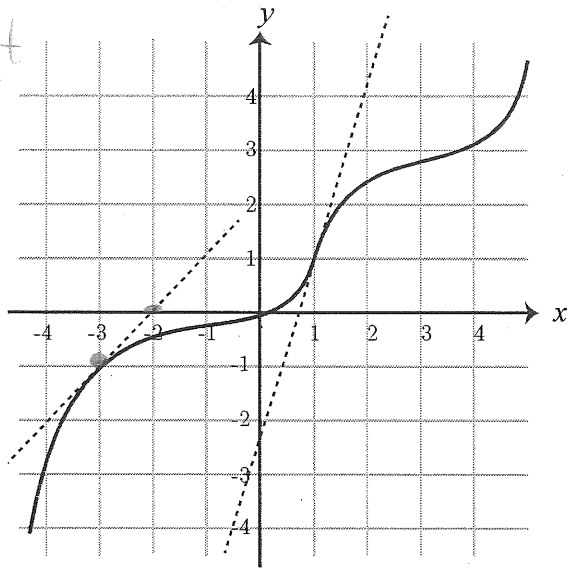
$2x-42=0$

$2x=42$

$x=21$

19. Determine the value of $f'(-3)$ from the graph of $f(x)$ given here:

$f'(-3)$ = slope of tangent line to graph of f at $x = -3$.



Possibilities:

(a) $f'(-3) = -3$

(b) $f'(-3) = 1$

(c) $f'(-3) = 0$

(d) $f'(-3) = -1$

(e) $f'(-3) = 3$

Use the points $(-3,-1)$ and $(-2,0)$ on the tangent line to find the slope

$f'(3) = \text{slope} = \frac{0-(-1)}{-2-(-3)} = \frac{1}{1} = 1$

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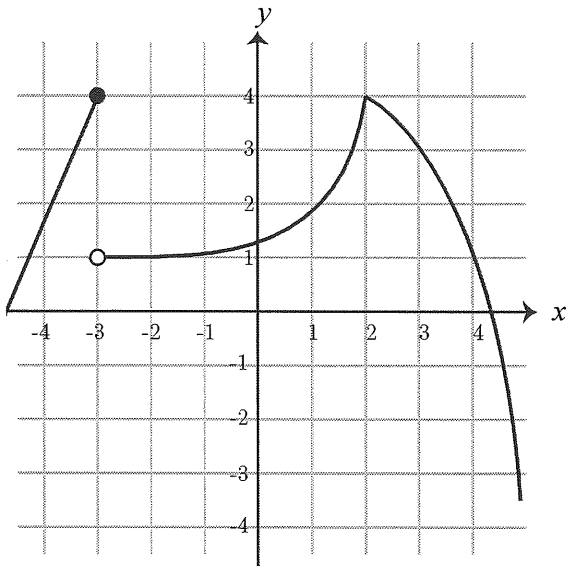
20. Determine the x values where the derivative is not defined (that is, the points where the function is not differentiable) on the function graphed here:



Possibilities:

(a) $x = -1$ and $x = 3$

(b) $x = -2$ and $x = 1$

(c) $x = -2$ and $x = 3$

(d) $x = -3$ and $x = 2$

(e) $x = -3$ and $x = 1$

The derivate is not defined at corners ($x=2$) and at points of discontinuity ($x=-3$).

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4.	a	**b**	c	d	e	**13.**	a	b	c	**d**	e
5.	a	b	c	**d**	e	**14.**	a	b	c	d	**e**
6.	**a**	b	c	d	e	**15.**	a	b	c	**d**	e
7.	a	b	**c**	d	e	**16.**	**a**	b	c	d	e
8.	a	b	c	d	**e**	**17.**	a	b	**c**	d	e
9.	a	b	c	**d**	e	**18.**	a	b	c	**d**	e
10.	a	**b**	c	d	e	**19.**	a	**b**	c	d	e
11.	a	b	**c**	d	e	**20.**	a	b	c	**d**	e

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