MA123 — Elem. Calculus Spring 2019
Exam 1 2019-02-07

Name: Solutions **Sec.:** _____

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

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GOOD LUCK!

3. a b ● d e

4. a ● c d e

5. ● b c d e

6. a b ● d e

7. a b c d ●

8. ● b c d e

9. a b c ● e

10. a b ● d e

11. a b c d ●

12. ● b c d e

13. a b ● d e

14. a ● c d e

15. a b ● d e

16. a b c d ●

17. a ● c d e

18. a b c ● e

19. a b ● d e

20. ● b c d e

For grading use:

Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(out of 100 points)

Spring 2019 Exam 1 Short Answer Questions

Write answers on this page. Your work must be clear and legible to be sure you will get full credit.

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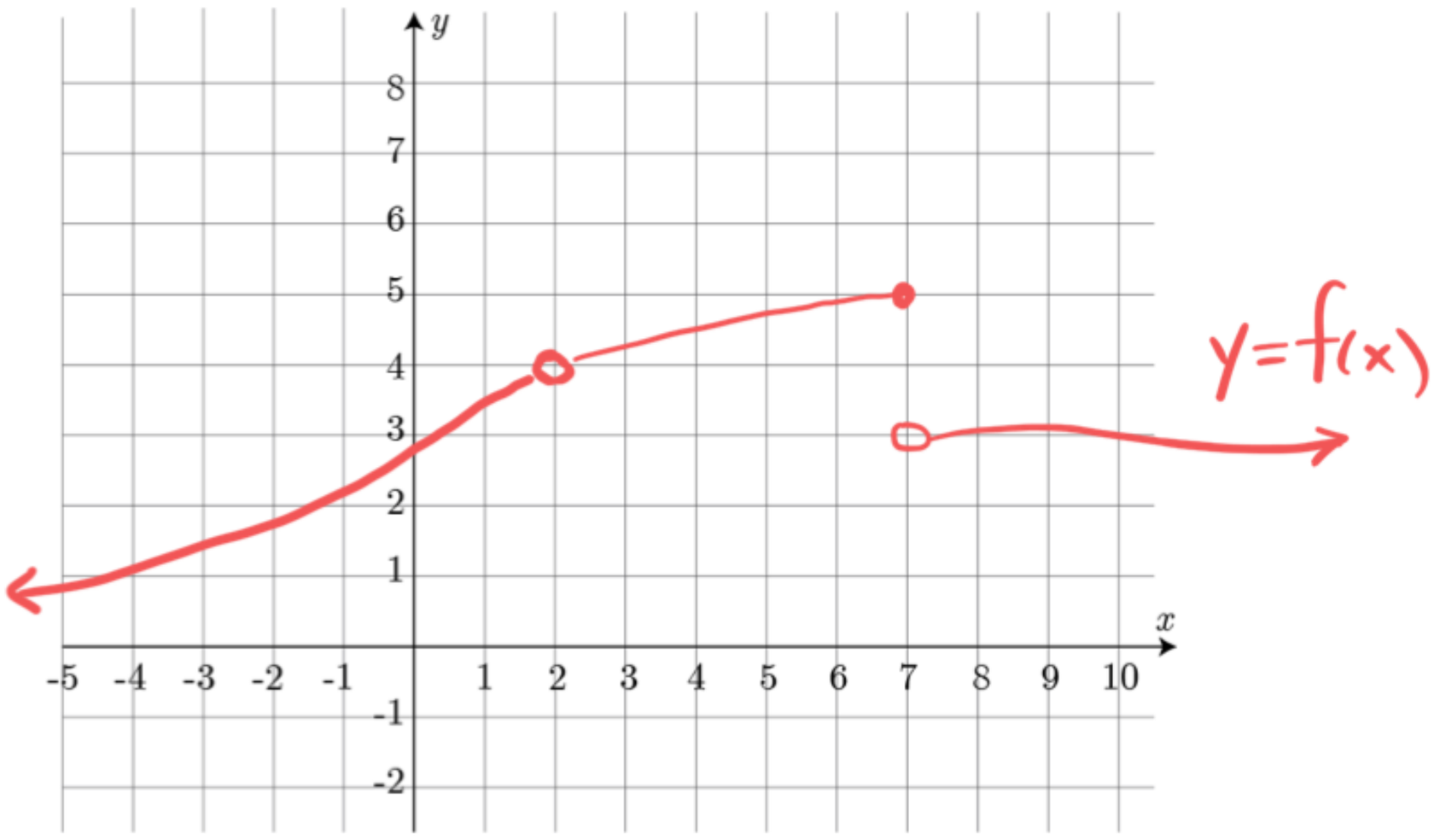
1. Sketch the graph of a single function $y = f(x)$ which satisfies the following properties:

   $\lim_{x \to 2} f(x) = 4$, $f(2)$ does not exist, $f(7) = 5$, $\lim_{x \to 7} f(x)$ does not exist, and $f(x)$ is continuous for all x except $x = 2$ and $x = 7$.



2. Let $f(x) = x^2 + 3x$. Find the **average rate of change** from $x = 5$ to $x = 5 + h$ and simplify your answer. Clearly circle your final answer.

AROC of $f(x)$ from $x=5$ to $x=5+h$

$$= \frac{f(5+h) - f(5)}{5+h-5} = \frac{(5+h)^2 + 3(5+h) - (5^2 + 3(5))}{h}$$

$$= \frac{25 + 10h + h^2 + 15 + 3h - 25 - 15}{h}$$

$$= \frac{10h + h^2 + 3h}{h} = \frac{h^2 + 13h}{h}$$

$$= \frac{h(h+13)}{h} = \boxed{h+13}$$

Name: _____

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

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3. The expression

$$\frac{x^{12}\,(2x)^3}{x^4}$$

can be simplified to which of the following?

Possibilities:

(a) $8x^7$

(b) $2x^{11}$

(c) $8x^{11}$

(d) $2x^9$

(e) $2x^7$

$\frac{x^{12}(2x)^3}{x^4} = \frac{x^{12}\cdot 2^3\cdot x^3}{x^4} = \frac{8x^{15}}{x^4}$

$= 8x^{11}$

---

4. The graph of $y = f(x)$ is shown below. The expression $f(a) = 0$ is true for which value(s) of a?

Possibilities:

(a) 0

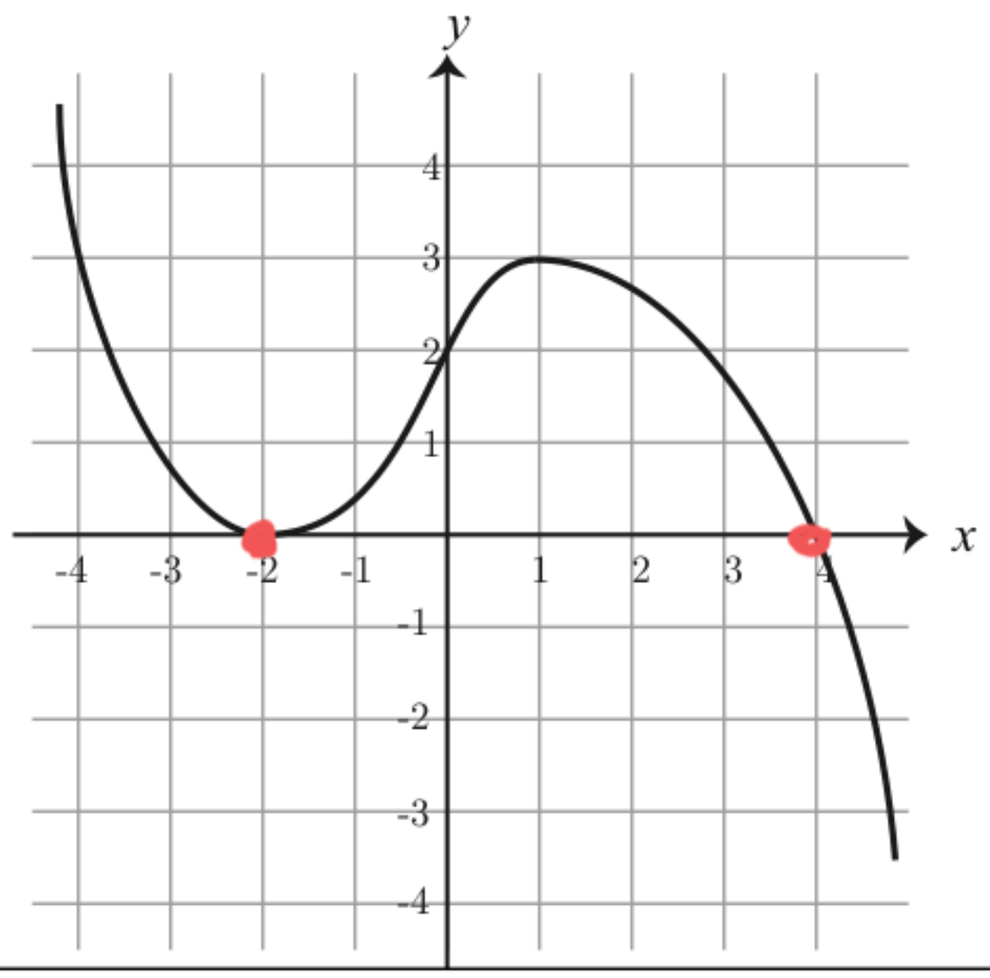
(b) $-2, 4$

(c) 2

(d) $0, 3$

(e) $-2, 1$

Find x values where $f(x)=y$ intersects the x-axis.



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5. If $h(t)$ represents the height of an object in feet above ground level at time t seconds and $h(t)$ is given by $h(t) = -16t^2 + 29t + 116$, find the time at which the speed of the object is zero.

Possibilities:

(a) (29/32) seconds

(b) 116 seconds

(c) (29/16) seconds

(d) (61/32) seconds

(e) (29/8) seconds

speed or velocity is the derivative

$h'(t) = -32t + 29$

set equal to zero and solve for t

$-32t + 29 = 0$

$-32t = -29$

$t = 29/32$

6. If $f(x) = \dfrac{8}{x+1}$ then choose the simplified form of $\frac{f(x+h)-f(x)}{h}$:

Possibilities:

(a) $-\dfrac{8 - h\,(x+1)^2}{(x+1)^2}$

(b) $\dfrac{16x + 16 + 8h}{(x+h+1)\,(x+1)\,(2x+h)}$

(c) $-\dfrac{8}{(x+h+1)\,(x+1)}$

(d) $-\dfrac{8}{(x+h+1)^2}$

(e) $\dfrac{8}{(x+h+1)\,(x+1)}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{8}{x+h+1} - \frac{8}{x+1}}{h}$$

$$= \frac{\frac{(x+1)}{(x+1)}\cdot\frac{8}{x+h+1} - \frac{(x+h+1)}{(x+h+1)}\cdot\frac{8}{x+1}}{h}$$

flip and multiply

$$= \frac{8(x+1) - 8(x+h+1)}{(x+1)(x+h+1)} \cdot \frac{1}{h}$$

$$= \frac{8x + 8 - 8x - 8h - 8}{(x+1)(x+h+1)} \cdot \frac{1}{h} = \frac{-8h}{h(x+1)(x+h+1)}$$

$$= \frac{-8}{(x+1)(x+h+1)}$$

7. The graph of $y = f(x)$ is shown below. Compute the average rate of change of $f(x)$ from $x = -4$ to $x = 2$.

Possibilities:

(a) $-\frac{7}{8}$

(b) $\frac{1}{6}$

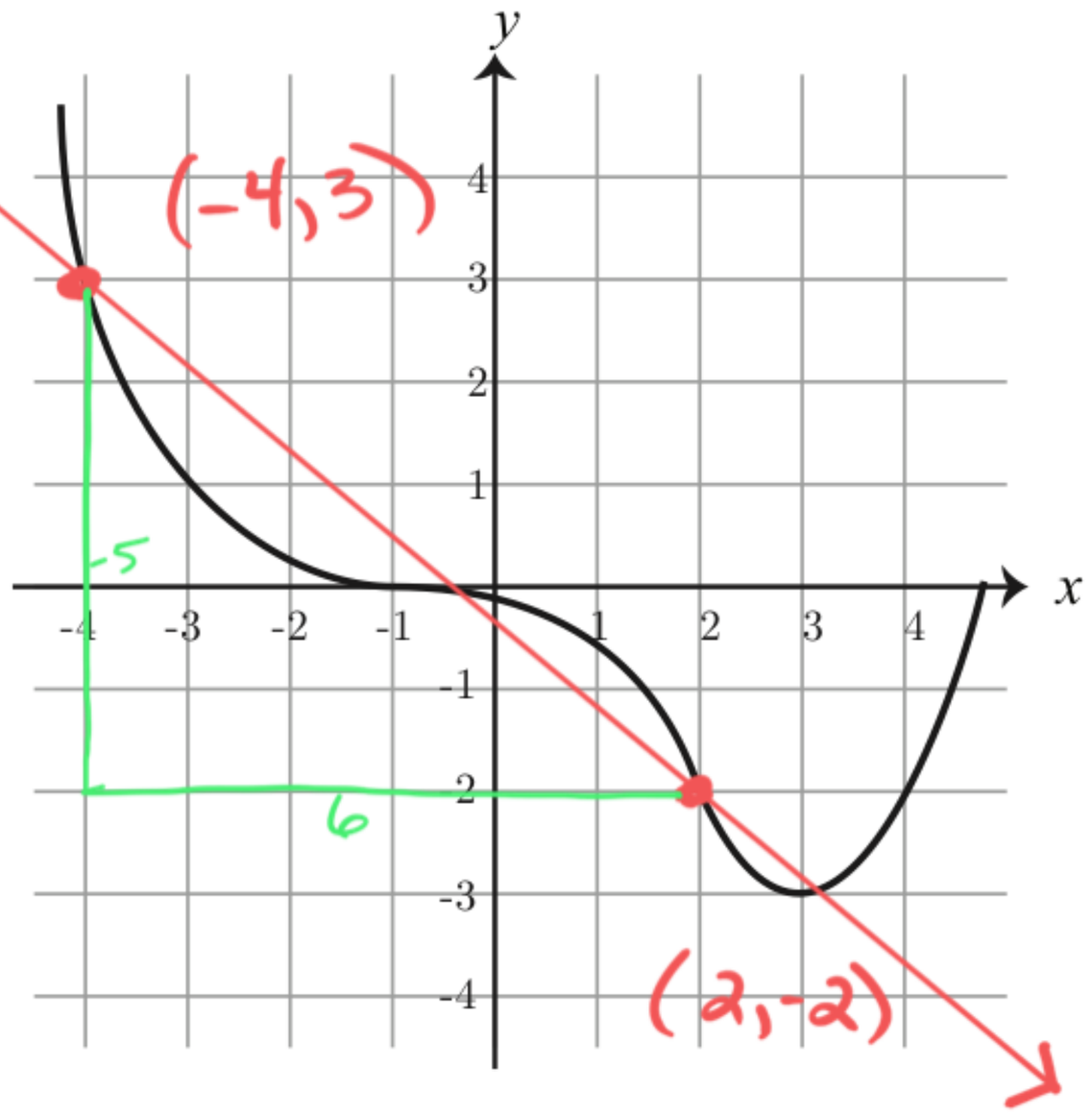
(c) $-\frac{6}{5}$

(d) $-\frac{2}{5}$

(e) $-\frac{5}{6}$

AROC is slope of this secant line

$$\text{Slope} = \frac{3-(-2)}{-4-2} = \frac{5}{-6}$$



8. Let $f(x) = x^5$. Find a value c between $x = 0$ and $x = 3$, so that the average rate of change of $f(x)$ from $x = 0$ to $x = 3$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$. You may use the fact that $f'(x) = 5x^4$.

Possibilities:

(a) $\frac{3}{\sqrt[4]{5}}$

(b) 81

(c) 3/2

(d) $\frac{3}{\sqrt{5}}$

(e) $\frac{5}{\sqrt[4]{3}}$

AROC of $f(x)$ from $x=0$ to $x=3$

$$= \frac{f(3)-f(0)}{3-0} = \frac{3^5-0^5}{3-0} = \frac{3^5}{3} = 3^4$$

IROC of $f(x)$ at $x=c$

$$= f'(c) = 5c^4$$

Set AROC = IROC and solve for c

$$3^4 = 5c^4$$

$$\Rightarrow \frac{3^4}{5} = c^4 \xrightarrow{4\text{th root}} \sqrt[4]{\frac{3^4}{5}} = c$$

$$\Rightarrow c = \frac{3}{\sqrt[4]{5}}$$

9. If $\lim_{x\to 13} f(x) = 3$ and $\lim_{x\to 13} g(x) = 5$, then what is the value of $\lim_{x\to 13} \dfrac{(x+7)(f(x)+1)}{g(x)}$?

Possibilities:

(a) $\dfrac{(13)(3)}{5}$

(b) $\dfrac{3}{5}$

(c) the limit is infinity or does not exist

(d) $\dfrac{(13+7)(3+1)}{5}$

(e) 0

$$= \frac{\lim_{x\to 13}(x+7)\cdot\lim_{x\to 13}(f(x)+1)}{\lim_{x\to 13} g(x)}$$

$$= \frac{\lim_{x\to 13}(x+7)\cdot\left[\lim_{x\to 13} f(x) + \lim_{x\to 13} 1\right]}{\lim_{x\to 13} g(x)}$$

$$= \frac{(13+7)(3+1)}{5}$$

10. Compute $\lim_{t\to 2} \dfrac{t^2+3t-10}{t^2-2t}$

Possibilities:

(a) 0

(b) 1

(c) $\frac{7}{2}$

(d) $\frac{9}{2}$

(e) The limit does not exist.

Plug in $t=2$ to get $\dfrac{2^2+3(2)-10}{2^2-2(2)} = \dfrac{0}{0}$ Do more work!

$$\lim_{t\to 2} \frac{(t-2)(t+5)}{t(t-2)}$$

$$= \lim_{t\to 2} \frac{t+5}{t} = \frac{2+5}{2} = \frac{7}{2}$$

11. Find the limit

$$\lim_{x\to 0}\left(\frac{15}{x}+\frac{3x-15}{x}\right)$$

Possibilities:

(a) This limit does not exist.

(b) 1

(c) 15

(d) 0

(e) 3

$= \lim_{x\to 0} \frac{15+3x-15}{x}$

$= \lim_{x\to 0} \frac{3x}{x}$

$= \lim_{x\to 0} 3$

$= 3$

---

12. Find the limit

$$\lim_{n\to\infty}\frac{(n+3)^2}{5n+13}$$

$= \lim_{n\to\infty} \frac{n^2+6n+9}{5n+13}$

Possibilities:

(a) The limit does not exist or approaches infinity

(b) $\frac{1}{13}$

(c) $\frac{1}{5}$

(d) $\frac{1}{18}$

(e) $\frac{9}{5}$

Infinite limit of rational function is the limit of the highest degree terms on top and bottom

$= \lim_{n\to\infty} \frac{n^2}{5n} = \lim_{n\to\infty} \frac{n}{5} = \infty$

$\Rightarrow$ DNE

---

13. For the function

$$f(x) = \begin{cases} 6x^2 + 9x + 3 & \text{if } x < -1 \\ \sqrt{x^2 + 9} & \text{if } -1 \le x < 2 \\ |8 + x| & \text{if } 2 \le x \end{cases}$$

← x=-6 is in this region

find $\lim_{x \to -6^+} f(x)$

Possibilities:

(a) 45

(b) $\sqrt{45}$

(c) 165

(d) 2

(e) $\sqrt{13}$

$= \lim_{x \to -6^+} 6x^2 + 9x + 3$

plug in x=-6 and see what happens.

$= 6(-6)^2 + 9(-6) + 3$

$= 165$

↑ Defined so this is the limit

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14. Find all values of x where the derivative is not defined for $f(x) = |x^2 - 11x + 18|$.

Possibilities:

(a) x = 0 and x = 18

(b) x = 2 and x = 9

(c) x = -11 only

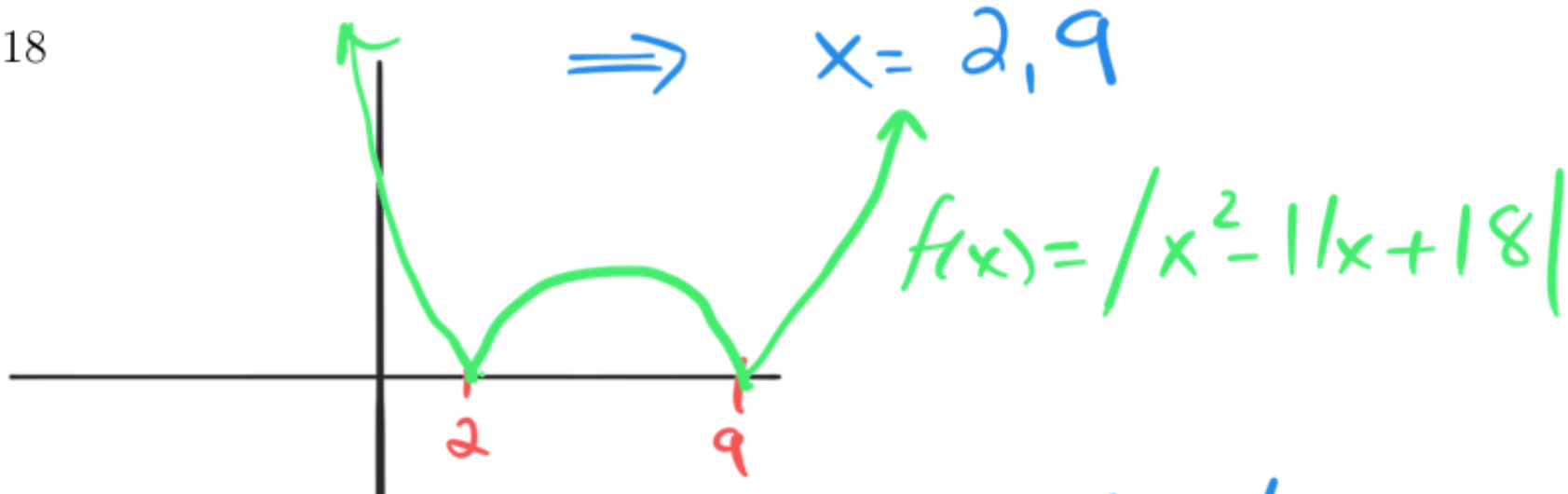
(d) x = -11 and x = 18

(e) x = 18 only

Set $x^2 - 11x + 18 = 0$ and solve for x

$(x-9)(x-2) = 0$

$\Rightarrow x = 2, 9$



corner point at x=2,9

⇒ not differentiable at x=2,9

15. Consider the function $f(x) = \begin{cases} Ax^2 & \text{if } x < 2 \\ 5 - Ax & \text{if } x \geq 2 \end{cases}$

Find a value of A so that the function is continuous at $x = 2$.

Possibilities:

(a) $\frac{1}{2}$

(b) $\frac{2}{3}$

(c) $\frac{5}{6}$

(d) 1

(e) $\frac{7}{6}$

Need $Ax^2 = 5 - Ax$ at $x = 2$ to be continuous

$\Rightarrow$ Solve $A(2)^2 = 5 - A(2)$

$4A = 5 - 2A$

$6A = 5$

$A = 5/6$

16. Determine the value of $f'(1)$ from the graph of $f(x)$ given here:

Possibilities:

(a) $f'(1) = 0$

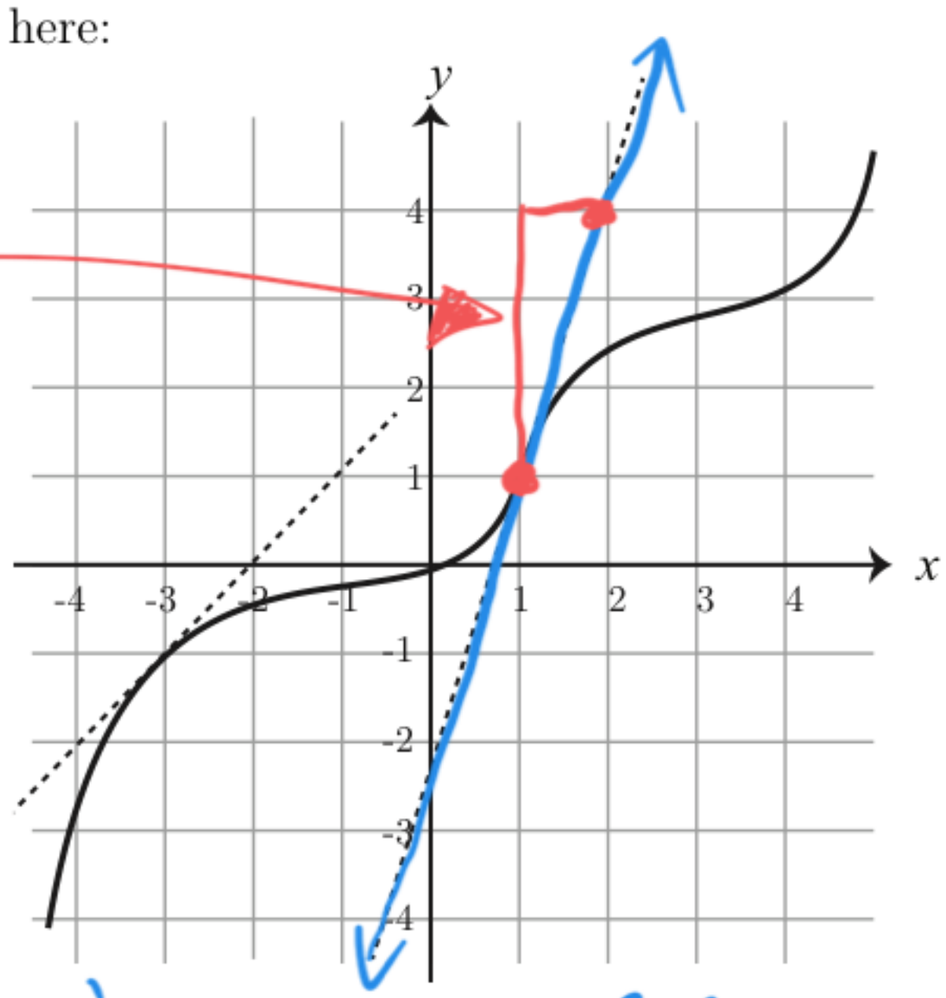
(b) $f'(1) = -1$

(c) $f'(1) = 1$

(d) $f'(1) = -3$

(e) $f'(1) = 3$

slope $= \frac{3}{1}$



$f'(1)$ is slope of tangent line to $f(x)$ at $x = 1$

17. For the function $f(x) = (x+4)^2$, find the equation of the tangent line to the graph of f at $x = 3$.

Possibilities:

(a) $y = 6x + 31$

(b) $y = 14x + 7$

(c) $y = 14x + 49$

(d) $y = x + 4$

(e) $y = 6x + 49$

$f(x) = (x+4)(x+4) = x^2 + 8x + 16$

Equation of tangent line is

$y - y_1 = m(x - x_1)$

$f(3)$ $f'(3)$ 3

$f(3) = 3^2 + 8(3) + 16$

$= 49$

$f'(x) = 2x + 8$

$\Rightarrow f'(3) = 2(3) + 8 = 14$

$y - 49 = 14(x - 3)$

$y - 49 = 14x - 42$

$y = 14x + 7$

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18. Consider the function $f(x) = x^2 + 8x + 1$. Its tangent line at $x = 3$ goes through the point $(9, y_1)$ where y_1 is:

Possibilities:

(a) -8

(b) 26

(c) 34

(d) 118

(e) 14

Find equation of tangent line

$y - y_1 = m(x - x_1)$

$f(3)$ $f'(3)$ 3

$f(3) = 3^2 + 8(3) + 1$

$= 34$

$f'(x) = 2x + 8$

$f'(3) = 2(3) + 8$

$= 14$

$y - 34 = 14(x - 3)$

$y - 34 = 14x - 42$

$y = 14x - 8$

Now plug in $x = 9$

$y = 14(9) - 8 = 118$

19. The graph of $y = f(x)$ is shown below. The function is **continuous**, except at $x =$

hole

Possibilities:

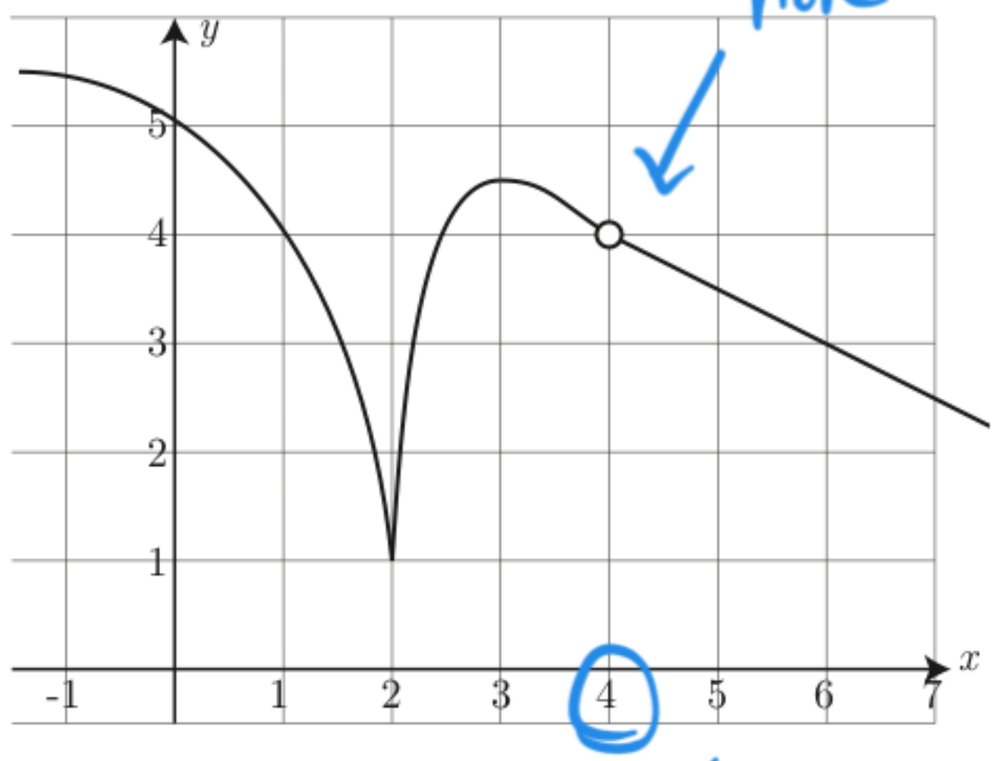
(a) x = 2 only

(b) x = 2 and x = 4

(c) x = 4 only

(d) x = 3 and x = 4

(e) x = 2, x = 3, and x = 4



Not continuous at the hole in the graph

x=4

20. The graph of $y = f(x)$ is shown below. The function is **differentiable**, except at $x =$

Possibilities:

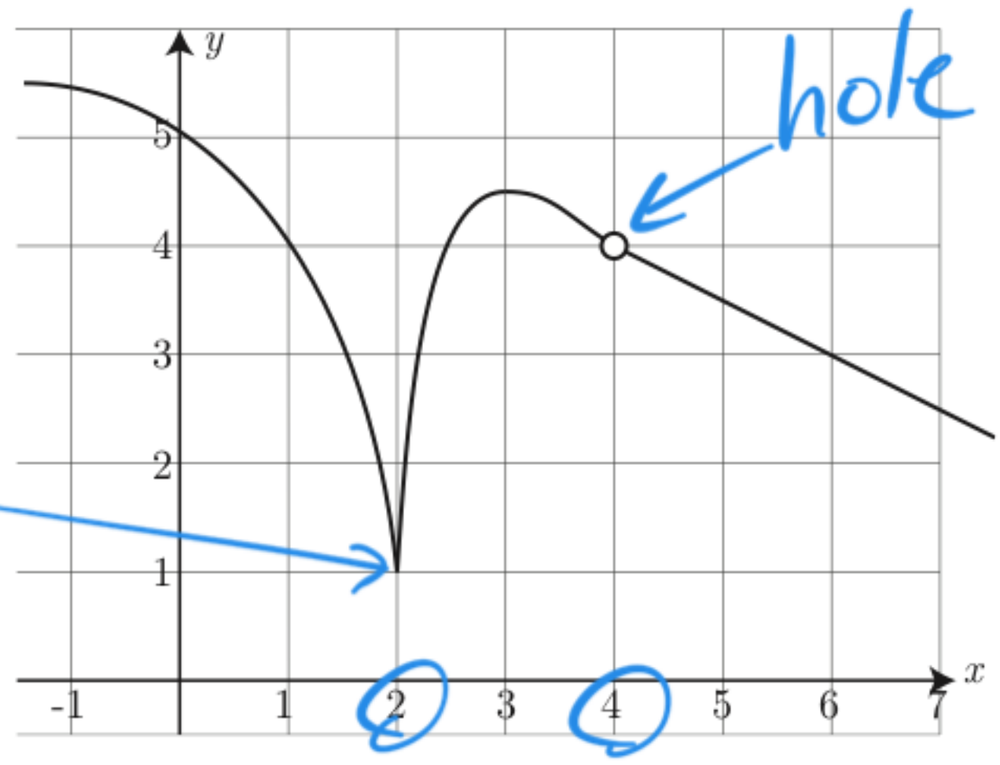
(a) x = 2 and x = 4

(b) x = 2, x = 3, and x = 4

(c) x = 4 only

(d) x = 3 and x = 4

(e) x = 2 only



Not differentiable at hole and corner point

x=2 x=4

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4.	a	**b**	c	d	e	**13.**	a	b	**c**	d	e
5.	**a**	b	c	d	e	**14.**	a	**b**	c	d	e
6.	a	b	**c**	d	e	**15.**	a	b	**c**	d	e
7.	a	b	c	d	**e**	**16.**	a	b	c	d	**e**
8.	**a**	b	c	d	e	**17.**	a	**b**	c	d	e
9.	a	b	c	**d**	e	**18.**	a	b	c	**d**	e
10.	a	b	**c**	d	e	**19.**	a	b	**c**	d	e
11.	a	b	c	d	**e**	**20.**	**a**	b	c	d	e

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