MA123 — Elem. Calculus Fall 2015
Exam 2 2015-10-22

Name: SOLUTIONS Sec.: _____

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name on this page, and at the top of page three.

GOOD LUCK!

3. (a) (b) (c) (d) (e)
4. (a) (b) (c) (d) (e)
5. (a) (b) (c) (d) (e)
6. (a) (b) (c) (d) (e)
7. (a) (b) (c) (d) (e)
8. (a) (b) (c) (d) (e)
9. (a) (b) (c) (d) (e)
10. (a) (b) (c) (d) (e)
11. (a) (b) (c) (d) (e)
12. (a) (b) (c) (d) (e)
13. (a) (b) (c) (d) (e)
14. (a) (b) (c) (d) (e)
15. (a) (b) (c) (d) (e)
16. (a) (b) (c) (d) (e)
17. (a) (b) (c) (d) (e)
18. (a) (b) (c) (d) (e)
19. (a) (b) (c) (d) (e)
20. (a) (b) (c) (d) (e)

For grading use:

Multiple Choice	Short Answer
(number right) (5 points each)	(out of 10 points)

Total	
	(out of 100 points)

Fall 2015 Exam 2 Short Answer Questions

*Write answers on this page. You must show **appropriate legible work** to be sure you will get full credit.*

4 pts 1. Find the **derivative** of $f(x) = x^3 \ln(8x+1)$. Do **NOT** simplify your answer.

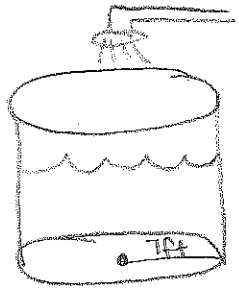
$f(x) = \underbrace{x^3}_{F(x)} \cdot \underbrace{\ln(8x+1)}_{G(x)}$ use product rule!

$f'(x) = F'(x)\,G(x) + F(x)\,G'(x)$

so $f'(x) = \underbrace{(3x^2)}_{\text{power rule}}(\ln(8x+1)) + x^3\underbrace{\left(\frac{1}{8x+1}\right)}_{\ln \text{ rule}}\underbrace{(8)}_{\text{derivative of inside}}$

$= \boxed{3x^2\ln(8x+1) + \frac{8x^3}{8x+1}}$

6 pts 2. A cylindrical water tank with its circular base parallel to the ground is being filled at the rate of 8 cubic feet per minute. The radius of the tank is 7 feet. How fast is the level of the water in the tank rising when the tank is half full?



$8 ft^3/min$

Volume of cylinder

$V = \pi r^2 h$ and $r = 7$ so

$V = \pi(7^2)h = 49\pi h$

differentiate!

$\frac{dV}{dt} = 49\pi \frac{dh}{dt}$

We're given $\frac{dV}{dt} = 8$ and need to find $\frac{dh}{dt}$

$8 = 49\pi \frac{dh}{dt}$

so $\frac{dh}{dt} = \boxed{\frac{8}{49\pi} \approx .052 \text{ ft/min}}$

Name: SOLUTIONS

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer on the cover page on this exam.

3. For the function $f(x) = x^3 + 2x^2 + 3x + 4$, find the equation of the tangent line to graph of f at $x = 2$.

$f'(x) = 3x^2 + 4x + 3$ Power rule

$f'(2) = 3(2)^2 + 4(2) + 3 = 23 \leftarrow$ Slope

$f(2) = 2^3 + 2(2)^2 + 3(2) + 4 = 26$ point $(2, 26)$

point slope form:

$y - 26 = 23(x - 2)$

$y - 26 = 23x - 46$

$y = 23x - 20$

Possibilities:

(a) $y = 23x - 20$

(b) $y = x^3 + 17$

(c) $y = 26x - 29$

(d) $y = 23x + 26$

(e) $y = 26$

4. Find the derivative, $f'(x)$, if $f(x) = \sqrt{4x^3 + 5x^2 + 6x + 2}$.

$f(x) = (4x^3 + 5x^2 + 6x + 2)^{1/2}$

Power rule & chain rule

$f'(x) = \frac{1}{2}(4x^3 + 5x^2 + 6x + 2)^{-1/2}(12x^2 + 10x + 6)$

Possibilities:

(a) $(1/2)(4x^3 + 5x^2 + 6x + 2)(12x^2 + 10x + 6)$

(b) $\sqrt{12x^2 + 10x + 6}$

(c) $(1/2)(4x^3 + 5x^2 + 6x + 2)^{1/2}$

(d) $\dfrac{\sqrt{12x^2 + 10x + 6}}{\sqrt{4x^3 + 5x^2 + 6x + 2}}$

(e) $(1/2)(4x^3 + 5x^2 + 6x + 2)^{-1/2}(12x^2 + 10x + 6)$

5. Find the derivative, $f'(x)$, if $f(x) = e^{4x^3+5x^2+6x+2}$.

Chain rule

$f'(x) = e^{4x^3 + 5x^2 + 6x + 2}(12x^2 + 10x + 6)$

Possibilities:

(a) $(12x^2 + 10x + 6)e^x$

(b) $\dfrac{12x^2 + 10x + 6}{4x^3 + 5x^2 + 6x + 2}$

(c) $(12x^2 + 10x + 6)e^{4x^3+5x^2+6x+2}$

(d) $e^{12x^2+10x+6}$

(e) $\ln(4x^3 + 5x^2 + 6x + 2)$

6. Suppose $F(x) = g(x) \cdot h(x+2)$. If $g(0) = 6$, $g'(0) = 9$, $h(0) = 8$, $h'(0) = 3$, $h(2) = 4$, and $h'(2) = 7$, find $F'(0)$.

Possibilities:

(a) 37

(b) 120

(c) 102

(d) 27

(e) 78

Product rule

$F'(x) = g'(x) \cdot h(x+2) + g(x) h'(x+2)\,(1)$

$F'(0) = g'(0) \cdot h(2) + g(0) h'(2)$

$= 9 \cdot 4 + 6 \cdot 7 = 36 + 42 = 78$

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7. Suppose $g(8) = -6$ and $g'(8) = -2$. Find $F'(8)$ if

$$F(x) = \frac{g(x)}{x^2}$$

Possibilities:

(a) $-\frac{1}{16}$

(b) $\frac{1}{128}$

(c) $-\frac{1}{2}$

(d) $-\frac{1}{128}$

(e) $-\frac{1}{4}$

quotient rule

$F'(x) = \dfrac{g'(x) \cdot x^2 - g(x) \cdot 2x}{(x^2)^2} = \dfrac{g'(x) \cdot x^2 - g(x) \cdot 2x}{x^4}$

$F'(8) = \dfrac{g'(8) \cdot 8^2 - g(8) \cdot 2(8)}{8^4} = \dfrac{-2(64) - (-6)(16)}{4096}$

$= \dfrac{-32}{4096} = \dfrac{-1}{128}$

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8. Suppose $F(x) = (g(x))^3 + 13$. If $g(2) = 9$, $g'(2) = 7$, and $g''(2) = 5$, then find $F'(2)$.

Possibilities:

(a) $(3)(9^2) + 13$

(b) $9^3 + 13$

(c) $(3)(9^2)(7)$

(d) $7^3 + 13$

(e) 5

Power rule, chain rule

$F'(x) = 3(g(x))^2 g'(x) + 0$

$F'(2) = 3(g(2))^2 \cdot g'(2)$

$= 3(9)^2 \cdot 7$

9. Suppose $F(x) = \ln(g(x))$. If $g(2) = 11$, $g'(2) = 7$, and $g''(2) = 5$, then find $F'(2)$.

Possibilities:

(a) $\ln(11)/7$

(b) $11/\ln(7)$

(c) $11/7$

(d) $\ln(5)$

(e) $7/11$

ln rule, chain rule

$$F'(x) = \frac{1}{g(x)} \cdot g'(x)$$

$$F'(2) = \frac{1}{g(2)} \cdot g'(2) = \frac{1}{11} \cdot 7 = \frac{7}{11}$$

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10. For the function $f(x) = \begin{cases} x^2 - 9 & x < 10 \\ \sqrt{x+4} & 10 \le x < 20 \\ x^3 - 8 & 20 \le x \end{cases}$, find the equation of the tangent line to the graph of f at $x = 26$.

$x = 26$ so 3rd line is what matters

Possibilities:

(a) $y = 667x - 4617$

(b) $y = 2028x - 35160$

(c) $y = \frac{1}{60}\sqrt{30}x + \frac{17}{30}\sqrt{30}$

(d) $y = \sqrt{30}x - \frac{419}{60}\sqrt{30}$

(e) $y = 52x - 685$

$f'(x) = 3x^2$ so $f'(26) = 3(26)^2 = 2028$ ← slope

$f(26) = (26)^3 - 8 = 17,568$ point $(26, 17568)$

point slope form

$y - 17568 = 2028(x - 26)$

$y - 17568 = 2028x - 52728$

$y = 2028x - 35160$

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11. Find the derivative, $f'(x)$, if $f(x) = (6 + 5x)\ln(4 + 8x)$.

Possibilities:

(a) $\frac{5}{4+8x}$

(b) $1/x$

(c) $(5)\ln(4+8x) + \frac{48+40x}{4+8x}$

(d) $\frac{13}{4+8x}$

(e) $5 + \frac{8}{4+8x}$

product rule

$$f'(x) = (5)\ln(4+8x) + (6+5x)\frac{1}{4+8x}(8)$$

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12. If $f(x) = 7x^4 + 5x^2 + 6x$ then find the second derivative $f''(x)$:

Possibilities:

(a) $84x^2 + 24$

(b) $28x^3 + 10x + 6$

(c) $28x^3 + 42x^2 + 38x + 18$

(d) $112x^4 + 20x^2$

(e) $84x^2 + 10$

Power rule (twice!)

$f'(x) = 28x^3 + 10x + 6$

$f''(x) = 84x^2 + 10$

13. If $f(x) = (14x + 36)^{27}$ then $f''(x) =$

Possibilities:

(a) $27^2\,(14)^{27}\,(14x + 36)$

(b) $27(26)\,(14x + 36)^{25}\,(14)^2$

(c) $27\,(14x + 36)^{26}$

(d) 0

(e) $27(26)14^{25}$

power rule with chain rule

$f'(x) = 27(14x + 36)^{26}(14)$

$f''(x) = 27(26)(14x + 36)^{25}(14)(14)$

14. Find the derivative, $f'(x)$, of $f(x) = \frac{1}{x^{10}}$

Possibilities:

(a) $1/(10\,x^{11})$

(b) $-10x^{-11}$

(c) $10x^9$

(d) $1/(10\,x^9)$

(e) $-10x^{-9}$

$f(x) = x^{-10}$ power rule

$f'(x) = -10\,x^{-11}$

15. How many years will it take an investment to triple in value if the interest rate is 5% compounded continuously?

$r = .05$

Possibilities:

(a) 12.21 years

(b) 13.73 years

(c) 15.69 years

(d) 18.31 years

(e) 21.97 years

$P = P_0 e^{rt}$

$\frac{3P_0}{P_0} = \frac{P_0 e^{.05t}}{P_0}$ Solve for t.

$3 = e^{.05t}$

$\ln 3 = \ln e^{.05t}$

$\ln 3 = .05t$

$t = \frac{\ln 3}{.05} \approx 21.97$ years

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16. The numbers of a bacteria in a culture doubles every 13 hours. How many hours will it take before 9 times the original number of bacteria is present?

Possibilities:

(a) $13 \ln(2)/\ln(9)$

(b) $\frac{13}{9}$

(c) $\frac{117}{2}$

(d) $13 \ln(9)/\ln(2)$

(e) $\frac{13}{2}$

$P(t) = P_0 e^{rt}$

doubles in 13 hrs → need to find r

$2P_0 = P_0 e^{r(13)}$

$2 = e^{13r}$

$\ln 2 = \ln e^{13r}$

$\ln 2 = 13r$

so $r = \frac{\ln 2}{13}$

how long before 9 times the original?

$9P_0 = P_0 e^{\frac{\ln 2}{13} t}$ Solve for t

$9 = e^{\frac{\ln 2}{13} t}$

$\ln 9 = \frac{\ln 2}{13} t$

$t = \frac{\ln 9}{\frac{\ln 2}{13}} = \frac{13 \ln 9}{\ln 2}$

17. A ladder 20 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 feet per second, how fast is the top of the ladder sliding down the wall (in feet per second) when the bottom of the ladder is 16 feet from the wall?

Possibilities:

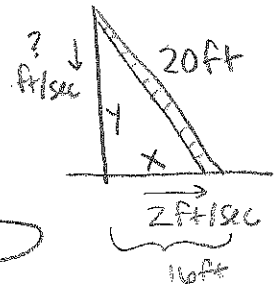
(a) $\frac{4}{3}$ feet per second

(b) $\frac{6}{5}$ feet per second

(c) $\frac{10}{3}$ feet per second

(d) $\frac{8}{3}$ feet per second

(e) $\frac{8}{5}$ feet per second



$x^2 + y^2 = 20^2$

differentiate!

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(16)(2) + 2(12)\frac{dy}{dt} = 0$

$64 + 24\frac{dy}{dt} = 0$

$24\frac{dy}{dt} = -64$

$\frac{dy}{dt} = \frac{-64}{24}$

negative = sliding down

if $x = 16$, then

$x^2 + y^2 = 20^2$

$16^2 + y^2 = 20^2$

$y^2 = 144$

$y = 12$

18. A farmer currently has harvested 270 bushels of turnip greens that are currently worth \$12.11 per bushel. The way things are going, he expects to be harvesting 4.00 bushels per day, and expects the price to be increasing at \$0.50 per bushel per day. What is the instantaneous rate of change (measured in dollars per day) of the total value of his turnip greens?

Possibilities:

(a) \$183.43 per day

(b) \$183.44 per day

(c) \$183.45 per day

(d) \$183.46 per day

(e) \$183.47 per day

$V = n \cdot p$ where V = total value, n = # of bushels, p = \$/bushel

differentiate!

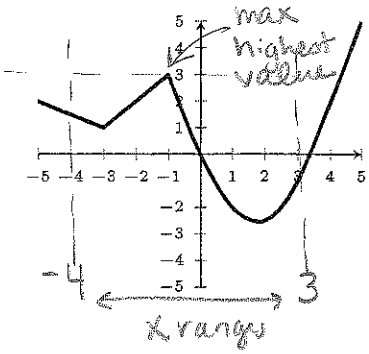
$\frac{dV}{dt} = \frac{dn}{dt} \cdot p + n \cdot \frac{dp}{dt}$

$\frac{dV}{dt} = (4) \cdot (12.11) + (270)(.50)$

$= 48.44 + 135$

$= 183.44$

19. The graph of $y = f(x)$ is shown below. The maximum value of $f(x)$ on the interval $[-4, 3]$ occurs at which x?



Possibilities:

(a) 7

(b) -1

(c) -3

(d) 2

(e) 0

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20. Find the maximum of $g(t) = (t+1)^2 - 4$ on the interval $[-2, 1]$

Possibilities:

(a) -3

(b) 2

(c) 0

(d) -4

(e) -1

$g(t)$ continuous ✓

then extreme values occur when $g'(t) = 0$

$g'(t) = 2(t+1)(1) - 0 = 2t + 2$

$2t + 2 = 0$

$2t = -2$

$t = -1$

Now check $t = -1$ and endpoints $t = -2, 1$

$g(-2) = (-2+1)^2 - 4 = 1 - 4 = -3$

$g(-1) = (-1+1)^2 - 4 = 0 - 4 = -4$

$g(1) = (1+1)^2 - 4 = 4 - 4 = 0$ ← max

Some Formulas

1. **Areas:**
   (a) Triangle $A = \frac{bh}{2}$
   (b) Circle $A = \pi r^2$
   (c) Rectangle $A = lw$
   (d) Trapezoid $A = \frac{h_1 + h_2}{2} b$

2. **Volumes:**
   (a) Rectangular Solid $V = lwh$
   (b) Sphere $V = \frac{4}{3}\pi r^3$
   (c) Cylinder $V = \pi r^2 h$
   (d) Cone $V = \frac{1}{3}\pi r^2 h$