MA123 — Elem. Calculus Exam 2	Fall 2016 2016-10-20	**Name:** SOLUTIONS **Sec.:** ________

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name on this page, and at the top of page three.

GOOD LUCK!

3.	a	b	**c**	d	e	**12.**	a	b	**c**	d	e
4.	**a**	b	c	d	e	**13.**	a	b	c	**d**	e
5.	a	**b**	c	d	e	**14.**	a	**b**	c	d	e
6.	**a**	b	c	d	e	**15.**	**a**	b	c	d	e
7.	a	**b**	c	d	e	**16.**	a	b	**c**	d	e
8.	a	b	c	d	**e**	**17.**	a	b	c	d	**e**
9.	**a**	b	c	d	e	**18.**	a	b	c	**d**	e
10.	a	b	c	**d**	e	**19.**	a	b	**c**	d	e
11.	**a**	b	c	d	e	**20.**	a	**b**	c	d	e

For grading use:

Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(out of 100 points)

Name: Last 4 digits of Student ID:

Fall 2016 Exam 2 Short Answer Questions

Write answers on this page. Your work must be clear and legible to be sure you will get full credit.

4 pts 1. Find the derivative of $f(x)=(3x+1)e^{10x+2}$. **Do NOT** simplify your answer.

Use The Product and Chain Rules

$$f'(x) = (3x+1)' \cdot e^{10x+2} + (3x+1)(e^{10x+2})'$$

$$= \boxed{3e^{10x+2} + (3x+1)(e^{10x+2})(10)}$$

6 pts 2. A circle is growing so that its area is increasing at a rate of 100 square feet per minute. At what rate is the radius of the circle changing when its radius is 8 feet? (Show steps clearly and circle your final answer.)

$$A(t) = \pi(r(t))^2$$

$$A'(t) = 2\pi r(t) \cdot r'(t) \quad \text{by chain rule}$$

$$\Rightarrow \frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

know $\frac{dA}{dt} = 100 \frac{ft^2}{min}$

- and -

$r = 8ft$

$$\Rightarrow 100 = 2\pi(8)\frac{dr}{dt}$$

$$\Rightarrow \frac{100}{16\pi} = \frac{dr}{dt}$$

radius is changing at a rate of $\frac{100}{16\pi} \frac{ft}{min}$

Name: SOLUTIONS

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer on the cover page on this exam.

3. For the function $f(x) = \ln(5x^2 + 6x + 2)$, find the equation of the tangent line to the graph of f at $x = 0$.

$y - y_1 = m(x - x_1) \Rightarrow y - f(0) = f'(0)(x-0)$

Possibilities:

(a) $y = \dfrac{2\,(5x+3)\,x}{5x^2+6x+2} + \ln(2)$

(b) $y = 2$

(c) $y = 3x + \ln(2)$

(d) $y = \ln(2)\,x + 6$

(e) $y = \frac{1}{3}x + \ln(2)$

$f'(x) = \dfrac{10x+6}{5x^2+6x+2} \Rightarrow f'(0) = \frac{6}{2} = 3$

$f(0) = \ln(2)$

$\Rightarrow y - \ln(2) = 3(x-0) \Rightarrow \boxed{y = 3x + \ln(2)}$

4. Find the derivative, $f'(x)$, if $f(x) = \sqrt[7]{2x^3 + 7x^2 + 8x + 1}$.

$f(x) = (2x^3 + 7x^2 + 8x + 1)^{1/7}$

Possibilities:

(a) $(1/7)(2x^3 + 7x^2 + 8x + 1)^{-6/7}(6x^2 + 14x + 8)$

(b) $\dfrac{\sqrt[7]{6x^2 + 14x + 8}}{\sqrt[7]{2x^3 + 7x^2 + 8x + 1}}$

(c) $(1/7)(2x^3 + 7x^2 + 8x + 1)^{-1/7}$

(d) $\sqrt[7]{6x^2 + 14x + 8}$

(e) $(1/7)(2x^3 + 7x^2 + 8x + 1)(6x^2 + 14x + 8)$

chain rule $\Rightarrow$

$f'(x) = \frac{1}{7}(2x^3+7x^2+8x+1)^{-6/7}(6x^2+14x+8)$

5. Find the derivative, $f'(x)$, if $f(x) = e^{2x+9} + 90x + 70$.

Possibilities:

(a) $\dfrac{2}{2x+9} + 90$

(b) $2e^{2x+9} + 90$

(c) $e^2 + 90$

(d) $\ln(2x + 9) + 160$

(e) $(2x + 9)e^{2x+8} + 90$

Chain rule

$\Rightarrow f'(x) = e^{2x+9}(2) + 90$

6. Suppose $F(x) = (x^3+3)g(x)$. If $g(1) = 4$ and $g'(1) = 8$, find $F'(1)$.

Possibilities: product rule

(a) 44

(b) 15

(c) 16

(d) 40

(e) 24

$$F'(x) = (x^3+3)'g(x) + (x^3+3)(g(x))'$$
$$= 3x^2 g(x) + (x^3+3)g'(x)$$
$$F'(1) = 3(1)^2 g(1) + (1^3+3)g'(1) = 3\cdot 4 + 4\cdot 8$$
$$= 44$$

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7. Suppose $g(4) = 7$ and $g'(4) = 6$. Find $F'(4)$ if

$$F(x) = \frac{x^3}{g(x)}$$

Quotient Rule

$$F'(x) = \frac{(x^3)'g(x) - x^3 g'(x)}{(g(x))^2}$$

Possibilities:

(a) $\frac{48}{49}$

(b) $-\frac{48}{49}$

(c) $-\frac{48}{7}$

(d) $\frac{6}{7}$

(e) -3

$$= \frac{3x^2 g(x) - x^3 g'(x)}{(g(x))^2}$$
$$F'(4) = \frac{3(4)^2 g(4) - 4^3 g'(4)}{(g(4))^2} = \frac{3\cdot 16\cdot 7 - 64\cdot 6}{49} = \frac{-48}{49}$$

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8. Suppose $H(x) = f(x^2 + g(x))$. If $g(2) = 7$, $g'(2) = 8$, $f'(12) = 10$, and $f'(11) = 17$, then find $H'(2)$.

Chain Rule

Possibilities:

(a) $(17)(12) + (11)(10)$

(b) 17

(c) $10(11)(4+17)$

(d) 10

(e) $17(4+8)$

$$H'(x) = f'(x^2+g(x))\cdot(2x+g'(x))$$
$$H'(2) = f'(2^2+g(2))\cdot(2(2)+g'(2))$$
$$= f'(11)\cdot(4+8)$$
$$= 17(4+8)$$

9. Suppose $F(x) = \ln(g(x))$. If $g(2) = 3$, $g'(2) = 11$, and $g''(2) = 7$, then find $F'(2)$.

$$F'(x) = \frac{1}{g(x)} \cdot g'(x)$$

$$F'(2) = \frac{1}{g(2)} \, g'(2) = \frac{1}{3} \cdot 11 = \frac{11}{3}$$

Possibilities:

(a) 11/3

(b) 3/11

(c) $\ln(3)/11$

(d) $\ln(7)$

(e) $3/\ln(11)$

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10. For the function $f(x) = \begin{cases} x^2 - 5 & x < 10 \\ \sqrt{x+9} & 10 \le x < 20 \\ x^3 - 6 & 20 \le x \end{cases}$, find the slope of the tangent line to the graph of f at $x = 18$.

use this piece

Possibilities:

$f'(x)$ between 10 & 20 is

$$\frac{1}{2}(x+9)^{-1/2}$$

$$f'(18) = \frac{1}{2}(27)^{-1/2} = \frac{1}{2\sqrt{27}} = \frac{\sqrt{27}}{54}$$

(a) 319

(b) 36

(c) 972

(d) $\frac{1}{54}\sqrt{27}$

(e) $\sqrt{27}$

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11. Find the derivative, $f'(x)$, if $f(x) = \ln(\ln(2 + 9x))$.

chain rule

$$f'(x) = \frac{1}{\ln(2+9x)} \cdot \frac{1}{2+9x} \cdot 9$$

Possibilities:

(a) $\frac{1}{\ln(2+9x)} \cdot \frac{9}{2+9x}$

(b) $\left(\frac{9}{2+9x}\right) e^{\ln(2+9x)}$

(c) $e^{\frac{9}{2+9x}}$

(d) $\frac{1}{\frac{9}{2+9x}}$

(e) $\frac{1}{\ln(\ln(2+9x))} \cdot \frac{1}{\ln(2+9x)} \cdot \frac{9}{2+9x}$

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12. If $f(x) = 5x^6 + 4x^4 + 2x^3$ then find the third derivative $f'''(x)$:

Possibilities:

(a) $30x^3 + 16x + 6$

(b) $30x^5 + 75x^4 + 116x^3 + 105x^2 + 52x + 11$

(c) $600x^3 + 96x + 12$

(d) $1080x^6 + 256x^4 + 54x^3$

(e) $150x^4 + 48x^2 + 12x$

$f'(x) = 30x^5 + 16x^3 + 6x^2$

$f''(x) = 150x^4 + 48x^2 + 12x$

$f'''(x) = 600x^3 + 96x + 12$

13. If $f(x) = e^{11x+37}$ then $f''(x) =$

Possibilities:

(a) $27^2\,(11)^{27}\,(11x+37)$

(b) $(11x+37)(11x+36)e^{11x+35}$

(c) $(11x+37)(11x+36)e^{11x+35} + 11e^{11x+36}$

(d) $11^2 e^{11x+37}$

(e) 0

$f'(x) = e^{11x+37}(11)$

$f''(x) = e^{11x+37}(11)(11)$

14. Find the derivative, $f'(x)$, of $f(x) = \frac{1}{x^{30}}$

Possibilities:

(a) $-30x^{-29}$

(b) $-30x^{-31}$

(c) $1/(30\,x^{29})$

(d) $1/(30\,x^{31})$

(e) $30x^{29}$

$f(x) = x^{-30}$

$\Rightarrow f'(x) = -30x^{-31}$

15. How many years will it take an investment to triple in value if the interest rate is 9% compounded continuously?

$\longrightarrow \quad P(t) = P_0 e^{rt}$

Possibilities:

(a) 12.21 years

(b) .122 years

(c) 1.31 years

(d) 7.70 years

(e) 14.37 years

triple $\Rightarrow \quad 3P_0 = P_0 e^{.09t}$

$3 = e^{.09t}$

$\ln(3) = .09t$

$\Rightarrow \frac{\ln(3)}{.09} = t$

$\Rightarrow t \approx 12.21$

16. The number of bacteria in a culture doubles every 5 hours. If we begin with 1000 cells, about how many cells do we have after 8 hours?

$P(t) = P_0 e^{rt}$

Possibilities:

(a) 3200 cells

(b) 625,000 cells

(c) 3031 cells

(d) 1542 cells

(e) 5800 cells

double $\Rightarrow \quad 2P_0 = P_0 e^{r \cdot 5}$

$\Rightarrow \quad 2 = e^{5r}$

$\ln(2) = 5r \Rightarrow r = \frac{\ln(2)}{5} \approx .139$

$P(8) = 1000 e^{.139(8)} \approx 3031$

17. A cylindrical water tank with its circular base parallel to the ground is being filled at the rate of 80 cubic feet per minute. The radius of the tank is 3 feet. How fast is the level of the water in the tank rising when the tank is half full?

Possibilities: doesnt matter

$V = \pi r^2 \cdot h$ radius is constant

(a) 1507.96 feet per minute

(b) 1.41 feet per minute

(c) 2261.95 feet per minute

(d) 4523.89 feet per minute

(e) 2.83 feet per minute

$\Rightarrow$ derivative of both sides with respect to time

$$\frac{dV}{dt} = (\pi r^2)\frac{dh}{dt}$$

$$80 = \pi(3)^2\frac{dh}{dt}$$

$$\frac{80}{9\pi} = 2.83 = \frac{dh}{dt}$$

18. Boyle's Law states that when a sample gas is compressed at a constant temperature, the pressure P and volume V satisfy the equation $PV = c$, where c is a constant. Suppose that at a certain instant the volume is 62 cubic centimeters, the pressure is 11 kPa, and the pressure is increasing at a rate of 3 kPa/min. At what rate is the volume decreasing at this instant?

Possibilities:

Both P and V are functions of time, c is constant

(a) $\frac{183}{11}$ cubic centimeters per minute

(b) $\frac{184}{11}$ cubic centimeters per minute

(c) $\frac{185}{11}$ cubic centimeters per minute

(d) $\frac{186}{11}$ cubic centimeters per minute

(e) 17 cubic centimeters per minute

$\Rightarrow$ By the product rule

$$\frac{dP}{dt}\cdot V + P\cdot\frac{dV}{dt} = 0$$

$$(3)(62) + (11)\cdot\frac{dV}{dt} = 0$$

$$\Rightarrow \frac{dV}{dt} = \frac{-186}{11}$$

decreasing

19. The graph of $y = f(x)$ is shown below. The minimum value of $f(x)$ on the interval $[-3, 4]$ occurs at which x?

Possibilities:

(a) 3

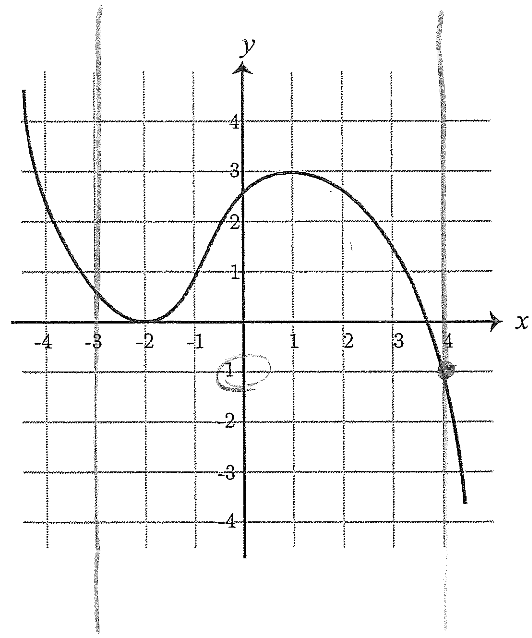
(b) -2

(c) 4

(d) -1

(e) 1

lowest point on graph of $y = f(x)$ between -3 and 4 is $(4, -1)$



minimum occurs at $x = 4$

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20. Find the minimum value of $g(t) = t^3 - 48t + 90$ on the interval $[-2, 5]$.

Possibilities:

(a) 218

(b) -38

(c) -25

(d) -36

(e) 178

$g'(t) = 3t^2 - 48 = 0$

$3t^2 = 48$

$t^2 = 16$

$t = \pm 4$

Need to check $g(-2)$, $g(4)$, and $g(5)$

$g(-2) = 178$

$g(4) = -38 \leftarrow$ min

$g(5) = -25$

Some Formulas

1. Areas:

(a) Triangle $A = \dfrac{bh}{2}$

(b) Circle $A = \pi r^2$

(c) Rectangle $A = lw$

(d) Trapezoid $A = \dfrac{h_1 + h_2}{2} b$

2. Volumes:

(a) Rectangular Solid $V = lwh$

(b) Sphere $V = \dfrac{4}{3}\pi r^3$

(c) Cylinder $V = \pi r^2 h$

(d) Cone $V = \dfrac{1}{3}\pi r^2 h$