MA123 — **Elem. Calculus**	Fall 2017	Name: Solutions	Sec.:
Exam 2	2017-10-19		

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and twenty multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name and section number on this page.

GOOD LUCK!

3.	c	12.	d
4.	d	13.	a
5.	e	14.	c
6.	c	15.	d
7.	b	16.	e
8.	e	17.	c
9.	b	18.	b
10.	a	19.	e
11.	c	20.	d

For grading use:

Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(out of 100 points)

Solutions

Fall 2017 Exam 2 Short Answer Questions

Write answers on this page. Your work must be clear and legible to be sure you will get full credit.

1. Find the derivative of $h(x) = e^{4x^3+9\ln x}$. **Do NOT** simplify your answer. Clearly circle your final answer.

$$\frac{d}{dx} e^{\text{stuff}} = e^{\text{stuff}} \cdot \frac{d}{dx}(\text{stuff})$$

$$\boxed{h'(x) = e^{4x^3+9\ln(x)} \cdot \left(12x^2 + \frac{9}{x}\right)}$$

2. Boyle's Law states that when a sample gas is compressed at a constant temperature, the pressure P and the volume V satisfy the equation $PV = c$, where c is a constant. Suppose that at a certain instant the volume is 700 cm³, the pressure is 300 kPa, and the volume is increasing at a rate of 4 cm³ per minute. At what rate is the pressure decreasing at this instant? (**Show steps clearly** and circle your final answer.)

Know

- $V = 700$
- $P = 300$
- $\frac{dV}{dt} = 4$

Find

$\frac{dP}{dt}$

$$PV = c$$

$$\frac{d}{dt}(PV) = \frac{d}{dt}(c)$$

product rule

$$P\frac{dV}{dt} + V\frac{dP}{dt} = 0$$

derivative of constant is 0

$$300(4) + 700\frac{dP}{dt} = 0$$

$$700\frac{dP}{dt} = -1200$$

$$\boxed{\frac{dP}{dt} = -\frac{1200}{700} = -\frac{12}{7} \approx -1.714 \ \frac{kPa}{min}}$$

Name: Solutions

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer on the cover page on this exam.

3. For the function $f(x) = 7x^3 + 8x^2 + 5x + 9$, find the equation of the tangent line to the graph of f at $x = 2$.

Need point and Slope

Possibilities:

(a) $y = 107x - 93$
(b) $y = 107$
(c) $y = 121x - 135$
(d) $y = 121x + 107$
(e) $y = x^3 + 17$

point: $x = 2$
$y = f(2) = 7(2)^3 + 8(2)^2 + 5(2) + 9 = 107$
$(2, 107)$

Slope: $f'(2)$
$f'(x) = 21x^2 + 16x + 5$
$f'(2) = 21(2)^2 + 16(2) + 5 = 121$

Eqn: $y - 107 = 121(x-2)$
$y - 107 = 121x - 242$
$\boxed{y = 121x - 135}$

4. Find the derivative, $f'(x)$, if $f(x) = \sqrt[5]{6x^3 + 7x^2 + 8x + 4}$. $= (6x^3 + 7x^2 + 8x + 4)^{\frac{1}{5}}$

Chain Rule!

Possibilities:

(a) $(1/5)(6x^3 + 7x^2 + 8x + 4)(18x^2 + 14x + 8)$
(b) $\sqrt[5]{18x^2 + 14x + 8}$
(c) $(1/5)(6x^3 + 7x^2 + 8x + 4)^{-1/5}$
(d) $(1/5)(6x^3 + 7x^2 + 8x + 4)^{-4/5}(18x^2 + 14x + 8)$
(e) $\dfrac{\sqrt[5]{18x^2 + 14x + 8}}{\sqrt[5]{6x^3 + 7x^2 + 8x + 4}}$

$f'(x) = \frac{1}{5}(6x^3 + 7x^2 + 8x + 4)^{-\frac{4}{5}} \cdot (18x^2 + 14x + 8)$

5. Find the derivative, $f'(x)$, if $f(x) = (80x + 70)\ln(9x + 5)$.

Product Rule! Chain Rule

Possibilities:

(a) $80 \cdot \frac{9}{9x+5}$
(b) $(80x + 70) \cdot \frac{1}{9x+5} + 80\ln(9x + 5)$
(c) $80\ln(9x + 5)$
(d) $9e^{9x+5} + 80$
(e) $(80x + 70) \cdot \frac{9}{9x+5} + 80\ln(9x + 5)$

$f'(x) = (80x+70) \cdot \frac{d}{dx}(\ln(9x+5)) + \ln(9x+5) \cdot \frac{d}{dx}(80x+70)$
$= (80x+70)\frac{1}{9x+5} \cdot (9) + \ln(9x+5)(80)$

6. Suppose $F(x) = (x+6)e^{g(x)}$. If $g(9) = 0$, and $g'(9) = 8$, find $F'(9)$.

Possibilities:

(a) 120

(b) 16

(c) 121

(d) 0

(e) 8

$F'(x) = (x+6)\frac{d}{dx}(e^{g(x)}) + e^{g(x)} \cdot \frac{d}{dx}(x+6)$

$F'(x) = (x+6)e^{g(x)} \cdot g'(x) + e^{g(x)}(1)$

$F'(9) = (9+6)e^{g(9)} \cdot g'(9) + e^{g(9)}$

$F'(9) = 15e^0(8) + e^0$

$= 120 + 1 = 121$

7. Suppose $g(7) = 6$ and $g'(7) = 5$. Find $F'(7)$ if

$$F(x) = \frac{x^2}{g(x)}$$

Quotient Rule

$F'(x) = \frac{g(x)\frac{d}{dx}(x^2) - x^2 g'(x)}{(g(x))^2}$

$F'(x) = \frac{g(x)(2x) - x^2 g'(x)}{(g(x))^2}$

$F'(7) = \frac{g(7)(14) - (7)^2 g'(7)}{(g(7))^2}$

$= \frac{6(14) - 49(5)}{(6)^2}$

$= \frac{84 - 245}{36}$

$= -\frac{161}{36}$

Possibilities:

(a) $-\frac{161}{6}$

(b) $-\frac{161}{36}$

(c) $-\frac{23}{7}$

(d) $\frac{161}{36}$

(e) $\frac{5}{6}$

8. Suppose $H(x) = \sqrt{f(x) + g(x)}$. If $f(8) = 4$, $f'(8) = 7$, $g(8) = 45$, and $g'(8) = 6$, find $H'(8)$.

Possibilities:

(a) $\frac{1}{14}$

(b) $\frac{1}{26}\sqrt{13}$

(c) $\sqrt{13}$

(d) $\frac{637}{2}$

(e) $\frac{13}{14}$

$H(x) = (f(x) + g(x))^{\frac{1}{2}}$ Chain Rule!

$H'(x) = \frac{1}{2}(f(x) + g(x))^{-\frac{1}{2}} \cdot (f'(x) + g'(x))$

$H'(8) = \frac{1}{2}(f(8) + g(8))^{-\frac{1}{2}} \cdot (f'(8) + g'(8))$

$= \frac{1}{2}(4 + 45)^{-\frac{1}{2}} \cdot (7 + 6)$

$= \frac{1}{2\sqrt{49}}(13) = \frac{13}{2(7)} = \frac{13}{14}$

9. Suppose $F(x) = \ln(g(x))$. If $g(2) = 11$, $g'(2) = 7$, and $g''(2) = 5$, then find $F'(2)$.

Possibilities:

(a) $\ln(11)/7$
(b) $7/11$
(c) $11/7$
(d) $11/\ln(7)$
(e) $\ln(5)$

$F'(x) = \frac{1}{g(x)} \cdot g'(x)$ ← Chain Rule!

$F'(2) = \frac{1}{g(2)} \cdot g'(2)$

$= \frac{1}{11}(7) = \frac{7}{11}$

$\frac{d}{dx}(\ln(\text{stuff})) = \frac{1}{\text{stuff}} \cdot \frac{d}{dx}(\text{stuff})$

---

10. For the function $f(x) = \begin{cases} x^2 - 9 & x < 10 \\ x^3 - 8 & 10 \le x < 20 \\ \sqrt{x+4} & 20 \le x \end{cases}$, find the slope of the tangent line to the graph of f at $x = 16$.

Possibilities:

(a) 768
(b) $\frac{1}{40}\sqrt{20}$
(c) 4088
(d) 247
(e) 32

Find $f'(16)$

Use middle piece for $x = 16$.

$f'(x) = 3x^2$ ← for middle piece

$f'(16) = 3(16)^2 = 3(256) = 768$

---

11. Find the derivative, $f'(x)$, if $f(x) = e^{\sqrt{9+5x}}$.

Possibilities:

(a) $e^{\left(\frac{\frac{5}{2}}{\sqrt{9+5x}}\right)}$

(b) $\frac{\frac{5}{2}\sqrt{9+5x}}{\sqrt{9+5x}}$

(c) $\left(\frac{\frac{5}{2}}{\sqrt{9+5x}}\right) e^{\sqrt{9+5x}}$

(d) $\left(\frac{\frac{5}{2}}{\sqrt{9+5x}}\right) e^{x}$

(e) $\ln(\sqrt{9+5x})$

Chain Rule!

$\frac{d}{dx}\left(e^{\text{stuff}}\right) = e^{\text{stuff}} \cdot \frac{d}{dx}(\text{stuff})$

$f'(x) = e^{\sqrt{9+5x}} \cdot \frac{d}{dx}\left(\sqrt{9+5x}\right)$

$= e^{\sqrt{9+5x}} \cdot \frac{d}{dx}\left((9+5x)^{\frac{1}{2}}\right)$

$= e^{\sqrt{9+5x}} \cdot \frac{1}{2}(9+5x)^{-\frac{1}{2}} \cdot (5)$

$= \frac{5e^{\sqrt{9+5x}}}{2\sqrt{9+5x}}$

---

12. If $f(x) = 2x^8 + 7x^2 + 8x$ then find the third derivative $f'''(x)$:

Possibilities:

(a) $672x^5 + 17x$

(b) $\dfrac{16x^7 + 14x + 8}{x^2}$

(c) $1024x^8 + 56x^2$

(d) $672x^5$

(e) $112x^6 + 14$

$f'(x) = 16x^7 + 14x + 8$

$f''(x) = 112x^6 + 14$

$f'''(x) = 112(6)x^5$

$= 672x^5$

---

13. If $f(x) = (14x + 36)^{27}$ then $f''(x) =$

Chain Rule!

Possibilities:

(a) $27(26)\,(14x + 36)^{25}\,(14)^2$

(b) 0

(c) $27\,(14x + 36)^{26}$

(d) $27^2\,(14)^{27}\,(14x + 36)$

(e) $27(26)14^{25}$

$f'(x) = 27(14x+36)^{26} \cdot (14) = 27(14)\,(14x+36)^{26}$

$f''(x) = 26(27)(14)\,(14x+36)^{25} \cdot (14)$

$= 26(27)(14)^2(14x+36)^{25}$

---

14. Find the derivative, $f'(x)$, of $f(x) = \frac{1}{x^{60}}$

$= x^{-60}$

Possibilities:

(a) $1/(60\,x^{59})$

(b) $-60x^{-59}$

(c) $-60x^{-61}$

(d) $60x^{59}$

(e) $1/(60\,x^{61})$

$f'(x) = -60\,x^{-61}$

15. If an amount of x dollars is invested at 5% interest compounded continuously, and at the end of 2 years the value of the investment is \$6000, find x.

Possibilities:

(a) \$4123.61

(b) \$5251.87

(c) \$5316.72

(d) \$5429.02

(e) \$6631.02

$P(t) = ae^{kt}$

$a =$ initial amount (x)

$k = 0.05$

$P(2) = xe^{0.05(2)}$

$6000 = xe^{0.1}$

$x = \frac{6000}{e^{0.1}} \approx \$5429.02$

---

16. The number of bacteria in a culture doubles every 7 hours. If we begin with 1000 cells, about how many cells do we have after 10 hours?

Possibilities:

(a) 1625 cells

(b) 2857 cells

(c) 16,807,000 cells

(d) 4804 cells

(e) 2692 cells

$P(t) = ae^{kt}$

Doubles after 7 hours $\Rightarrow$ $2000 = 1000e^{k(7)}$

$2 = e^{7k}$

$7k = \ln(2)$

$k = \frac{\ln(2)}{7}$

We want to find $P(10)$.

$P(10) = 1000e^{\frac{\ln(2)}{7}(10)}$

≈ 2692 cells

17. A circle is growing so its area is increasing at a rate of 91 square feet per minute. At what rate is the radius changing when its radius is 5 feet?

Possibilities:

(a) 910π feet per minute

(b) $\frac{10\pi}{91}$ feet per minute

(c) $\frac{91}{10\pi}$ feet per minute

(d) $\frac{91}{25\pi}$ feet per minute

(e) $\frac{91}{5\pi}$ feet per minute

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 91, \quad r = 5$

$91 = 2\pi(5)\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{91}{10\pi}$

---

18. It is estimated that the annual advertising revenue received by a certain newspaper will be

$$R(x) = 0.5x^2 + 9x + 195$$

thousand dollars when its circulation is x thousand. The circulation of the paper is currently 17000 and is increasing at a rate of 2000 papers per year. At what rate will the annual advertising revenue be increasing with respect to time 3 years from now?

Possibilities:

(a) \$52000.00 per year

(b) \$64000.00 per year

(c) \$492.50 per year

(d) \$5500.00 per year

(e) \$45900.00 per year

Now

$x = 17$ (17000 papers)

In 3 years

6000 more papers

$\Rightarrow$ 23000 papers so $x = 23$

$\frac{dx}{dt} = 2$

~~$R'(x) = 2(0.5)x + 9$~~

~~$R'(x) = x + 9$~~

$\frac{dR}{dt} = 2(0.5)x\frac{dx}{dt} + 9\frac{dx}{dt}$

$\frac{dR}{dt} = x\frac{dx}{dt} + 9\frac{dx}{dt}$

$= 23(2) + 9(2) = 46 + 18 = 64$ $\Rightarrow$ revenue is increasing by \$64000 /year

this in thousands

19. The graph of $y = f(x)$ is shown below. The minimum value of $f(x)$ on the interval $[-3, 4]$ occurs at which x?

Possibilities:

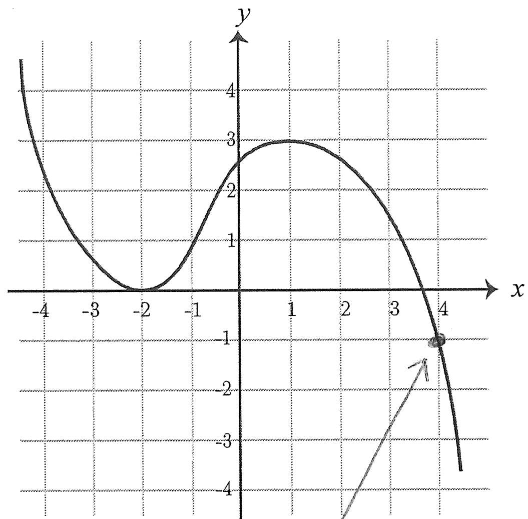
(a) 3

~~(b) −1~~

(c) −2

(d) 1

(e) 4



Smallest y-value on $[-3,4]$.

The minimum value is -1.

$\boxed{\text{It occurs at } x=4}$.

20. Find the minimum value of $g(t) = t^3 - 48t + 70$ on the interval $[-2, 5]$.

Possibilities:

(a) −45

(b) 198

(c) 158

(d) −58

(e) −36

$g'(t) = 3t^2 - 48$

Set $g'(t)=0$ to find crit. pts.

$0 = 3t^2 - 48$

$3t^2 = 48$

$t^2 = 16$

$t = \pm 4$

Plug crit. pts. in our interval and the endpoints into g:

$g(-2) = (-2)^3 - 48(-2) + 70 = -8 + 96 + 70 = 158$

$g(4) = (4)^3 - 48(4) + 70 = 64 - 192 + 70 = -58$

$g(5) = (5)^3 - 48(5) + 70 = 125 - 240 + 70 = -45$

Some Formulas

1. **Areas:**

   (a) Triangle $A = \dfrac{bh}{2}$

   (b) Circle $A = \pi r^2$

   (c) Rectangle $A = lw$

   (d) Trapezoid $A = \dfrac{h_1 + h_2}{2} b$

2. **Volumes:**

   (a) Rectangular Solid $V = lwh$

   (b) Sphere $V = \frac{4}{3}\pi r^3$

   (c) Cylinder $V = \pi r^2 h$

   (d) Cone $V = \frac{1}{3}\pi r^2 h$