MA123 — Elem. Calculus Spring 2016
Exam 2 2016-3-10

Name: SOLUTIONS Sec.: _____

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name on this page, and at the top of page three.

GOOD LUCK!

3. (a) (b) (c) **(d)** (e)
4. (a) **(b)** (c) (d) (e)
5. (a) (b) (c) **(d)** (e)
6. (a) (b) **(c)** (d) (e)
7. **(a)** (b) (c) (d) (e)
8. (a) **(b)** (c) (d) (e)
9. (a) (b) (c) **(d)** (e)
10. (a) (b) **(c)** (d) (e)
11. (a) (b) (c) **(d)** (e)
12. (a) **(b)** (c) (d) (e)
13. (a) (b) **(c)** (d) (e)
14. **(a)** (b) (c) (d) (e)
15. (a) (b) **(c)** (d) (e)
16. (a) (b) (c) (d) **(e)**
17. **(a)** (b) (c) (d) (e)
18. (a) (b) (c) (d) **(e)**
19. **(a)** (b) (c) (d) (e)
20. (a) (b) (c) **(d)** (e)

For grading use:

Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(out of 100 points)

Spring 2016 Exam 2 Short Answer Questions

Write answers on this page. You must show appropriate legible work to be sure you will get full credit.

4 pts 1. Find the **derivative** of $f(x)=\frac{2x+4}{9x+7}$. **Do NOT simplify your answer.**

(quotient rule)

$$f'(x)=\frac{(9x+7)(2x+4)'-(2x+4)(9x+7)'}{(9x+7)^2}$$

$$f'(x)=\frac{(9x+7)(2)-(2x+4)(9)}{(9x+7)^2}$$

6 pts 2. The demand function q for a certain product is given by $q=3000e^{-0.004p}$, where p denotes the price of the product. If the item is currently selling for \$450 per unit, and the quantity supplied is decreasing at a rate of 80 units per week, find the rate at which the price of the product is changing.

$$q=3000e^{-.004p}$$

take derivative with respect to time

$$\frac{dq}{dt}=3000e^{-.004p}(-.004)\frac{dp}{dt}$$

$$\frac{dq}{dt}=-12e^{-.004p}\frac{dp}{dt}$$

plug in what we know

$$-80=-12e^{-.004(450)}\frac{dp}{dt}$$

$$-80=-12e^{-1.8}\frac{dp}{dt}$$

$$\frac{80}{12}=e^{-1.8}\frac{dp}{dt}$$

$$\frac{dp}{dt}=\frac{80/12}{e^{-1.8}}\approx 40.3309$$

the price is increasing by \$40.33 per week.

Name: SOLUTIONS

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer on the cover page on this exam.

3. For the function $f(x) = x^3 + 2x^2 + 3x + 4$, find the equation of the tangent line to graph of f at $x = 1$.

$f(1) = 1^3 + 2(1)^2 + 3(1) + 4 = 10$ point $(1, 10)$

$f'(x) = 3x^2 + 4x + 3$

$f'(1) = 3(1)^2 + 4(1) + 3 = 10$ slope $m = 10$

tangent line $y - 10 = 10(x - 1)$

$y = 10(x-1) + 10$

$y = 10x$

Possibilities:

(a) $y = 10x + 10$

(b) $y = 10$

(c) $y = x^3 + 17$

(d) $y = 10x$

(e) 6

4. Find the derivative, $f'(x)$, if $f(x) = \sqrt[5]{4x^3 + 5x^2 + 6x + 2}$.

$f(x) = (4x^3 + 5x^2 + 6x + 2)^{1/5}$

$f'(x) = \frac{1}{5}(4x^3 + 5x^2 + 6x + 2)^{-4/5}(12x^2 + 10x + 6)$

"Power rule" "Chain rule"

Possibilities:

(a) $(1/5)(4x^3 + 5x^2 + 6x + 2)(12x^2 + 10x + 6)$

(b) $(1/5)(4x^3 + 5x^2 + 6x + 2)^{-4/5}(12x^2 + 10x + 6)$

(c) $(1/5)(4x^3 + 5x^2 + 6x + 2)^{-1/5}$

(d) $\sqrt[5]{12x^2 + 10x + 6}$

(e) $\dfrac{\sqrt[5]{12x^2 + 10x + 6}}{\sqrt[5]{4x^3 + 5x^2 + 6x + 2}}$

5. Find the derivative, $f'(x)$, if $f(x) = e^{4x^3+5x^2+6x+2}$.

$(e^{f(x)})' = e^{f(x)} \cdot f'(x)$

$f'(x) = e^{4x^3+5x^2+6x+2}(4x^3 + 5x^2 + 6x + 2)'$

$= e^{4x^3+5x^2+6x+2}(12x^2 + 10x + 6)$

Possibilities:

(a) $(12x^2 + 10x + 6)e^x$

(b) $\dfrac{12x^2 + 10x + 6}{4x^3 + 5x^2 + 6x + 2}$

(c) $e^{12x^2+10x+6}$

(d) $(12x^2 + 10x + 6)e^{4x^3+5x^2+6x+2}$

(e) $\ln(4x^3 + 5x^2 + 6x + 2)$

6. Suppose $F(x) = (x^3 + 6)g(x)$. If $g(1) = 8$ and $g'(1) = 3$, find $F'(1)$.

Possibilities:

(a) 9

(b) 17

(c) 45

(d) 56

(e) 65

Product rule

$F'(x) = (x^3+6)\,g'(x) + g(x)(x^3+6)'$

$= (x^3+6)\,g'(x) + g(x)(3x^2)$

$F'(1) = (1^3+6)\,g'(1) + g(1)(3\cdot 1^2)$

$= 7(3) + (8)(3)$

$= 21 + 24 = 45$

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7. Suppose $g(6) = 5$ and $g'(6) = 4$. Find $F'(6)$ if

$$F(x) = \frac{x^2}{g(x)}$$

Possibilities:

(a) $-\frac{84}{25}$

(b) $\frac{84}{25}$

(c) $-\frac{7}{3}$

(d) $-\frac{84}{5}$

(e) $\frac{4}{5}$

Quotient rule

$F'(x) = \dfrac{g(x)(x^2)' - (x^2)g'(x)}{g(x)^2} = \dfrac{g(x)\cdot(2x) - x^2\cdot g'(x)}{g(x)^2}$

$F'(6) = \dfrac{g(6)(2\cdot 6) - (6^2)g'(6)}{g(6)^2} = \dfrac{5(12) - 36(4)}{25} = \dfrac{-84}{25}$

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8. Suppose $F(x) = (g(x) + 13)^3$. If $g(2) = 9$, $g'(2) = 7$, and $g''(2) = 5$, then find $F'(2)$.

Possibilities:

(a) $(3)(20^2)$

(b) $(3)(22)^2(7)$

(c) $(3)(9^2) + 13$

(d) $(3)(7)^2$

(e) $9^3 + 13$

$F'(x) = 3(g(x)+13)^2 \cdot g'(x)$ power rule with chain rule

$F'(2) = 3(g(2)+13)^2 \cdot g'(2)$

$= 3(9+13)^2(7)$

$= 3(22)^2(7)$

$= 10,164$

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9. Suppose $F(x) = \ln(g(x))$. If $g(2) = 11$, $g'(2) = 7$, and $g''(2) = 5$, then find $F'(2)$.

Possibilities:

(a) $\ln(11)/7$

(b) $11/\ln(7)$

(c) $11/7$

(d) $7/11$

(e) $\ln(5)$

$F'(x) = \frac{1}{g(x)} \cdot g'(x)$ Chain rule

$F'(2) = \frac{1}{g(2)} \cdot g'(2) = \frac{1}{11} \cdot 7 = \frac{7}{11}$

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10. For the function $f(x) = \begin{cases} x^2 - 9 & x < 10 \\ \sqrt{x+4} & 10 \le x < 20 \\ x^3 - 8 & 20 \le x \end{cases}$, find the slope of the tangent line to the graph of f at $x = 16$.

Possibilities:

(a) $\sqrt{20}$

(b) 768

(c) $\frac{1}{40}\sqrt{20}$

(d) 247

(e) 32

at $x = 16 \rightarrow f(x) = (x+4)^{1/2}$ Use power rule

$f'(x) = \frac{1}{2}(x+4)^{-1/2}(1)$

$f'(16) = \frac{1}{2}(16+4)^{-1/2} = \frac{1}{2} \cdot \frac{1}{\sqrt{20}} = \frac{1}{2\sqrt{20}}$

$= \frac{1}{2\sqrt{20}} \cdot \frac{\sqrt{20}}{\sqrt{20}} = \frac{\sqrt{20}}{40}$

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11. Find the derivative, $f'(x)$, if $f(x) = (6+5x)\ln(4+8x)$.

Possibilities:

(a) $(5)\ln(4+8x) + \frac{6+5x}{x}$

(b) $\frac{13}{4+8x}$

(c) $\frac{5}{4+8x}$

(d) $(5)\ln(4+8x) + \frac{48+40x}{4+8x}$

(e) $5 + \frac{8}{4+8x}$

Product rule

$f'(x) = (6+5x)[\ln(4+8x)]' + (6+5x)'\ln(4+8x)$

$= (6+5x)\left(\frac{1}{4+8x}\right)(8) + (5)\ln(4+8x)$

$= \frac{8(6+5x)}{4+8x} + 5\ln(4+8x)$

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12. If $f(x) = 7x^4 + 5x^2 + 6x$ then find the second derivative $f''(x)$:

Possibilities:

(a) $84x^2 + 24$

(b) $84x^2 + 10$

(c) $28x^3 + 10x + 6$

(d) $112x^4 + 20x^2$

(e) $28x^3 + 42x^2 + 38x + 18$

$f'(x) = 28x^3 + 10x + 6$

$f''(x) = 84x^2 + 10$

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13. If $f(x) = e^{14x+36}$ then $f''(x) =$

Possibilities:

(a) $(14x + 36)(14x + 35)e^{14x+34} + 14e^{14x+35}$

(b) $27^2\,(14)^{27}\,(14x + 36)$

(c) $14^2 e^{14x+36}$

(d) $(14x + 36)(14x + 35)e^{14x+34}$

(e) 0

$f'(x) = e^{14x+36}(14)$

$= 14e^{14x+36}$

$f''(x) = 14e^{14x+36}(14)$

$= 196e^{14x+36}$

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14. Find the derivative, $f'(x)$, of $f(x) = \frac{1}{x^{10}}$

Possibilities:

(a) $-10x^{-11}$

(b) $1/(10\,x^{11})$

(c) $10x^9$

(d) $1/(10\,x^9)$

(e) $-10x^{-9}$

$f(x) = x^{-10}$ rewrite

$f'(x) = -10x^{-11}$ Power rule

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15. If \$5000 dollars is invested at 4% interest compounded continuously, what is the value of the investment at the end of 7 years?

Possibilities:

(a) \$1400.00

(b) \$3778.92

(c) \$6615.65

(d) \$36428.38

(e) \$82223.23

$P = P_0 e^{rt}$

$P = 5000\, e^{.04(7)}$

$P = 5000\, e^{.28}$

$P = 6615.6490$

$= \$6615.65$

16. The number of a bacteria in a culture triples every 13 hours. How many hours will it take before 10 times the original number of bacteria is present?

Possibilities:

(a) $13 \ln(3)/\ln(10)$

(b) $\frac{13}{10}$

(c) $\frac{130}{3}$

(d) $\frac{13}{3}$

(e) $13 \ln(10)/\ln(3)$

$P = P_0 e^{rt}$

$3P_0 = P_0 e^{r(13)}$ ← triples every 13 hrs. find the rate

$3 = e^{13r}$

$\ln 3 = \ln e^{13r}$

$\ln 3 = 13r$

$r = \frac{\ln 3}{13}$

how many hours to get 10 times

$P = P_0 e^{rt}$

$10P_0 = P_0 e^{\left(\frac{\ln 3}{13}\right)t}$

$10 = e^{\left(\frac{\ln 3}{13}\right)t}$

$\ln 10 = \ln e^{\left(\frac{\ln 3}{13}\right)t}$

$\ln 10 = \left(\frac{\ln 3}{13}\right)t$

$t = \frac{\ln 10}{\left(\frac{\ln 3}{13}\right)} = \frac{13 \ln 10}{\ln 3}$

17. Two birds leave the same tree at different times, one traveling due East, and the other traveling due North. At 2pm the eastbound bird is traveling at 25 mph and is 40 miles from the tree, while the northbound bird is traveling at 20 mph and is 30 miles from the tree. At what rate is the distance between the birds increasing?

Possibilities:

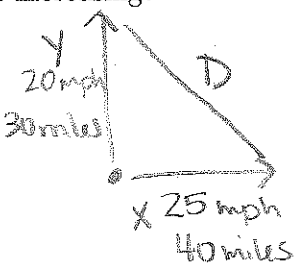
(a) 32 mph

(b) 50 mph

(c) 45 mph

(d) $5\sqrt{41}$ mph

(e) 3200 mph



$30^2+40^2=D^2$

$D=50$

$x^2+y^2=D^2$

take derivative with respect to time

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2D\frac{dD}{dt}$

plug in what we know

$2(40)(25)+2(30)(20)=2(50)\frac{dD}{dt}$

$3200=100\frac{dD}{dt}$

$32=\frac{dD}{dt}$

18. Boyle's Law states that when a sample gas is compressed at a constant temperature, the pressure P and volume V satisfy the equation $PV = c$, where c is a constant. Suppose that at a certain instant the volume is 49 cubic centimeters, the pressure is 11 kPa, and the pressure is increasing at a rate of 3 kPa/min. At what rate is the volume decreasing at this instant?

Possibilities:

(a) 13 cubic centimeters per minute

(b) $\frac{144}{11}$ cubic centimeters per minute

(c) $\frac{145}{11}$ cubic centimeters per minute

(d) $\frac{146}{11}$ cubic centimeters per minute

(e) $\frac{147}{11}$ cubic centimeters per minute

$PV=c$

take derivative with respect to time

$P\frac{dV}{dt}+\frac{dP}{dt}V=0$

plug in what we know

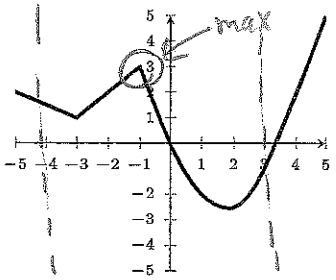
$(11)\frac{dV}{dt}+(3)(49)=0$

$11\frac{dV}{dt}=-147$

$\frac{dV}{dt}=\frac{-147}{11}$

volume is decreasing at $\frac{147}{11}$ cm^3/min

19. The graph of $y = f(x)$ is shown below. The maximum value of $f(x)$ on the interval $[-4, 3]$ occurs at which x?



Possibilities:

(a) -1

(b) -3

(c) 7

(d) 2

(e) 0

20. Find the minimum of $g(t) = -(t+2)^2 + 7$ on the interval $[-3, 0]$

Possibilities:

(a) 7

(b) 6

(c) -2

(d) 3

(e) 0

$g'(t) = -2(t+2)$

$g'(t) = 0$ when $t = -2$

Check $t = -3, 0, -2$

$g(-3) = -(-3+2)^2 + 7 = -1 + 7 = 6$

$g(0) = -(0+2)^2 + 7 = -4 + 7 = 3 \leftarrow$ min

$g(-2) = -(-2+2)^2 + 7 = 0 + 7 = 7$

Some Formulas

1. Areas:

(a) Triangle $A = \frac{bh}{2}$

(b) Circle $A = \pi r^2$

(c) Rectangle $A = lw$

(d) Trapezoid $A = \frac{h_1 + h_2}{2} b$

2. Volumes:

(a) Rectangular Solid $V = lwh$

(b) Sphere $V = \frac{4}{3}\pi r^3$

(c) Cylinder $V = \pi r^2 h$

(d) Cone $V = \frac{1}{3}\pi r^2 h$