MA123 — Elem. Calculus Spring 2017
Exam 2 2017-03-09

Name: SOLUTIONS Sec.: _____

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name on this page, and at the top of page three.

GOOD LUCK!

3.	a	b	**c**	d	e	12.	a	b	c	**d**	e
4.	**a**	b	c	d	e	13.	**a**	b	c	d	e
5.	a	b	c	d	**e**	14.	a	b	**c**	d	e
6.	a	b	c	**d**	e	15.	a	**b**	c	d	e
7.	a	**b**	c	d	e	16.	a	b	c	**d**	e
8.	**a**	b	c	d	e	17.	a	b	**c**	d	e
9.	a	**b**	c	d	e	18.	**a**	b	c	d	e
10.	**a**	b	c	d	e	19.	a	**b**	c	d	e
11.	a	b	c	d	**e**	20.	a	b	c	**d**	e

For grading use:

Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(total 100 points)

Name: Last 4 digits of Student ID:

Spring 2017 Exam 2 Short Answer Questions

Write answers on this page. You must show appropriate legible steps to be sure you will get full credit.

4 pts 1. Find the **derivative** of $f(x)=(3x+10)\ln(11x+2)$. Do **NOT** simplify your answer.

Product Rule and Chain Rule

$$f'(x)=(3x+10)(\ln(11x+2))'+(3x+10)'\ln(11x+2)$$

$$=\boxed{(3x+10)\frac{1}{11x+2}\cdot 11+3\ln(11x+2)}$$

6 pts 2. A cylindrical water tank with its circular base parallel to the ground is being filled at the rate of 17 cubic feet per minute. The radius of the tank is 8 feet. How fast is the level of the water in the tank rising when the tank is half full?

$V=\pi r^2 h$ (Plug in $r=8$)

$V=\pi(8)^2 h$ (The radius is constant for a cylinder)

$V=64\pi h$

Take derivative of both sides $\left(\frac{d}{dt}\right)$

$$\frac{dV}{dt}=64\pi\frac{dh}{dt}$$

Plug in $\frac{dV}{dt}=17$

$$17=64\pi\frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt}=\frac{17}{64\pi}\ \frac{ft}{min}}$$

Name: SOLUTIONS

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer on the cover page on this exam.

3. For the function $f(x) = 4x^3 + 9x^2 + 2x + 5$, find the equation of the tangent line to the graph of f at $x = 2$.

slope $= f'(2)$

$f'(x) = 12x^2 + 18x + 2$

$f'(2) = 12(2)^2 + 18(2) + 2$

$= 48 + 36 + 2$

$= 86$

point

$x = 2$

$y = 4(2)^3 + 9(2)^2 + 2(2) + 5$

$= 77$

equation: $y - 77 = 86(x - 2)$

$y = 86x - 172 + 77 = 86x - 95$

Possibilities:

(a) $y = 77$

(b) $y = 86x + 77$

(c) $y = 86x - 95$

(d) $y = 77x - 68$

(e) $y = x^3 + 17$

4. Find the derivative, $f'(x)$, if $f(x) = \sqrt[3]{9x^2 + 7x + 1}$.

$= (9x^2 + 7x + 1)^{\frac{1}{3}}$ Chain Rule

$f'(x) = \frac{1}{3}(9x^2 + 7x + 1)^{-\frac{2}{3}}(18x + 7)$

Possibilities:

(a) $(1/3)(9x^2 + 7x + 1)^{-2/3}(18x + 7)$

(b) $(1/3)(9x^2 + 7x + 1)^{-1/3}$

(c) $\sqrt[3]{18x + 7}$

(d) $\dfrac{\sqrt[3]{18x + 7}}{\sqrt[3]{9x^2 + 7x + 1}}$

(e) $(1/3)(9x^2 + 7x + 1)(18x + 7)$

5. Find the derivative, $f'(x)$, if $f(x) = \ln(5x + 2) + 70x + 90$.

$f'(x) = \frac{1}{5x+2} \cdot 5 + 70$

$= \frac{5}{5x+2} + 70$

Possibilities:

(a) $\ln(5x + 2) + 160$

(b) $\dfrac{1}{\ln(5x + 2)} \cdot \dfrac{5}{5x + 2} + 70$

(c) $(5x + 2)e^{5x+1} + 70$

(d) $5e^{5x+2} + 70$

(e) $\dfrac{5}{5x + 2} + 70$

6. Suppose $F(x) = g(x)e^{6x}$. If $g(0) = 5$ and $g'(0) = 4$, find $F'(0)$.

Product Rule and Chain Rule

Possibilities:

(a) 24

(b) 4

(c) 15

(d) 34

(e) 9

$F'(x) = g(x)(e^{6x})' + g'(x)e^{6x}$

$= g(x) \cdot 6e^{6x} + g'(x)e^{6x}$

$F'(0) = g'(0) \cdot 6e^{0} + g'(0)e^{0}$

$= 5(6)(1) + 4(1)$

$= 34$

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7. Suppose $g(4) = -10$ and $g'(4) = 9$. Find $F'(4)$ if

$$F(x) = \frac{g(x)}{x^2}$$

Use Quotient Rule

Possibilities:

(a) $\frac{9}{4}$

(b) $\frac{7}{8}$

(c) $-\frac{7}{8}$

(d) $\frac{7}{2}$

(e) 14

$F'(x) = \frac{x^2 g'(x) - g(x)(2x)}{(x^2)^2}$

$F'(4) = \frac{(4)^2 g'(4) - g(4)(2)(4)}{(4^2)^2}$

$= \frac{16(9) - (-10)(8)}{256}$

$= \frac{7}{8}$

---

8. Suppose $F(x) = (g(x))^5 + 13$. If $g(2) = 9$, $g'(2) = 3$, and $g''(2) = 7$, then find $F'(2)$.

Use Chain Rule

Possibilities:

(a) $(5)(9^4)(3)$

(b) $3^5 + 13$

(c) $9^5 + 13$

(d) 7

(e) $(5)(9^4) + 13$

$F'(x) = 5(g(x))^4 \cdot g'(x)$

$F'(2) = 5(g(2))^4 \cdot g'(2)$

$= 5(9)^4(3)$

---

9. Suppose $F(x) = \ln(g(x))$. If $g(2) = 11$, $g'(2) = 3$, and $g''(2) = 7$, then find $F'(2)$.

Possibilities:

(a) $\ln(7)$

(b) $3/11$

(c) $11/\ln(3)$

(d) $\ln(11)/3$

(e) $11/3$

Use Chain Rule

$F'(x) = \frac{1}{g(x)} \cdot g'(x)$

$F'(2) = \frac{g'(2)}{g(2)} = \frac{3}{11}$

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10. For the function $f(x) = \begin{cases} x^2 - 4 & x < 10 \\ x^3 - 5 & 10 \le x < 20 \\ \sqrt{x+6} & 20 \le x \end{cases}$, find the slope of the tangent line to the graph of f at $x = 18$.

↑ use this piece

Possibilities:

(a) 972

(b) $\frac{1}{48}\sqrt{24}$

(c) 36

(d) 320

(e) 5827

Slope $= f'(18)$

$f'(x) = 3x^2$

Plug in $x = 18$

$f'(18) = 3(18)^2$

---

11. Find the derivative, $f'(x)$, if $f(x) = \ln(\ln(5+2x))$.

Use Chain Rule

Possibilities:

(a) $\dfrac{1}{\dfrac{2}{5+2x}}$

(b) $e^{\frac{2}{5+2x}}$

(c) $\dfrac{1}{\ln(\ln(5+2x))} \cdot \dfrac{1}{\ln(5+2x)} \cdot \dfrac{2}{5+2x}$

(d) $\left(\dfrac{2}{5+2x}\right) e^{\ln(5+2x)}$

(e) $\dfrac{1}{\ln(5+2x)} \cdot \dfrac{2}{5+2x}$

$f'(x) = \frac{1}{\ln(5+2x)} \cdot \frac{1}{5+2x} \cdot 2$

$= \frac{1}{\ln(5+2x)} \cdot \frac{2}{5+2x}$

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12. If $f(x) = 2x^7 + 9x^2 + 7x$ then find the third derivative $f'''(x)$:

$f'(x) = 14x^6 + 18x + 7$

$f''(x) = 84x^5 + 18$

$f'''(x) = 420x^4$

Possibilities:

(a) $14x^6 + 42x^5 + 70x^4 + 70x^3 + 42x^2 + 32x + 18$

(b) $84x^5 + 18$

(c) $\dfrac{14x^6 + 18x + 7}{x^2}$

(d) $420x^4$

(e) $686x^7 + 72x^2$

---

13. If $f(x) = (11x + 37)^{27}$ then $f''(x) =$

$f'(x) = 27(11x+37)^{26} \cdot 11$

Use Chain Rule:

$f''(x) = 27(26)(11x+37)^{25} \cdot 11^2$

Possibilities:

(a) $27(26)\,(11x + 37)^{25}\,(11)^2$

(b) $27^2\,(11)^{27}\,(11x + 37)$

(c) 0

(d) $27(26)11^{25}$

(e) $27\,(11x + 37)^{26}$

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14. Find the derivative, $f'(x)$, of $f(x) = \frac{1}{x^{60}}$ $= x^{-60}$

$f'(x) = -60x^{-61}$

Possibilities:

(a) $-60x^{-59}$

(b) $60x^{59}$

(c) $-60x^{-61}$

(d) $1/(60\,x^{59})$

(e) $1/(60\,x^{61})$

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15. If \$7000 dollars is invested at 5% interest compounded continuously, what is the value of the investment at the end of 4 years?

Possibilities:

(a) \$1400.00

(b) \$8549.82

(c) \$5731.12

(d) \$29435.59

(e) \$51723.39

$P(t) = P_0 e^{rt}$

$P(t) = 7000 e^{.05(4)}$

$= 8549.819$

16. A bacteria culture starts with 2000 bacteria and doubles after 13 hours. If we express the number of bacteria after t hours as $y(t) = a \cdot e^{kt}$, find the value of k.

Possibilities:

(a) $\ln(2) / \ln(13)$

(b) $2000 / \ln(2)$

(c) $13 / \ln(2)$

(d) $\ln(2) / 13$

(e) 2000

$y(t) = a \cdot e^{kt}$

doubles after 13 hours $\Rightarrow 4000 = 2000 e^{k(13)}$

$2 = e^{13k}$

$13k = \ln(2)$

$k = \frac{1}{13} \ln(2)$

17. Two trains leave the same station at different times, one traveling due East, and the other traveling due North. At 2pm the eastbound train is traveling at 60 mph and is 400 miles from the station, while the northbound train is traveling at 45 mph and is 300 miles from the station. At what rate is the distance between the trains increasing?

Possibilities:

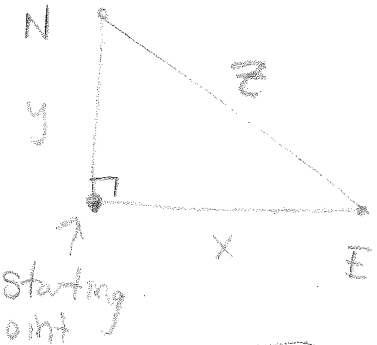
(a) 6 mph

(b) 500 mph

(c) 75 mph

(d) 75000 mph

(e) 105 mph



$z^2 = x^2 + y^2$

Differentiate:

$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$2(500)\frac{dz}{dt} = 2(400)(60) + 2(300)(45)$

$\frac{dz}{dt} = 75 \frac{mi}{hr}$

$300^2 + 400^2 = z^2$

$z = 500$

Know

- $\frac{dx}{dt} = 60$
- $\frac{dy}{dt} = 45$

Find $\frac{dz}{dt}$

18. A stock is increasing in value at a rate of \$0.75 per share per hour. An investor is buying shares of the stock at a rate of 3 shares per hour. How fast is the value of the investor's stock growing when the price is \$13.52 per share and the investor owns 230 shares? Hint: The total value of the stock is given by $V = NP$ where N is the number of shares currently owned and P is the current price.

Possibilities:

(a) \$213.06 per hour

(b) \$700.14 per hour

(c) \$2.25 per hour

(d) \$3111.85 per hour

(e) \$214.06 per hour

Know

$\frac{dP}{dt} = 0.75$

$\frac{dN}{dt} = 3$

$V = NP$

Differentiate and use Product Rule:

$\frac{dV}{dt} = N\frac{dP}{dt} + \frac{dN}{dt}P$

$\frac{dV}{dt} = 230(0.75) + 3(13.52)$

$= \$213.06$ per hour

19. The graph of $y = f(x)$ is shown below. What is the maximum value of $f(x)$ on the interval $[-3, 4]$?

Possibilities:

(a) 4

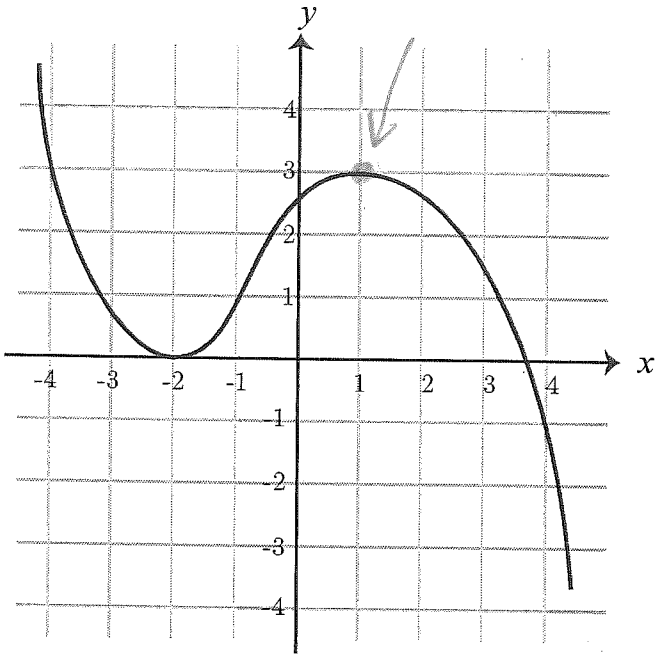
(b) 3

(c) -1

(d) 0

(e) 2

The maximum value is the largest y-value on the interval $[-3, 4]$, the highest y-value is 3



20. Find the maximum value of $f(x) = -x^2 - 12x + 54$ on the interval $[-10, 10]$.

Possibilities:

(a) 80

(b) 74

(c) 100

(d) 90

(e) -166

Find critical points by setting $f'(x) = 0$

$f'(x) = -2x - 12$

$0 = -2x - 12$

$2x = -12$

$x = -6$

Plug in critical points and endpoints into $f(x)$

$f(-10) = -(-10)^2 - 12(-10) + 54 = 74$

$f(-6) = -(-6)^2 - 12(-6) + 54 = 90$ ← maximum value

$f(10) = -(10)^2 - 12(10) + 54 = -166$ ← minimum value

Some Formulas

1. Areas:

(a) Triangle $A = \frac{bh}{2}$

(b) Circle $A = \pi r^2$

(c) Rectangle $A = lw$

(d) Trapezoid $A = \frac{h_1 + h_2}{2} b$

2. Volumes:

(a) Rectangular Solid $V = lwh$

(b) Sphere $V = \frac{4}{3}\pi r^3$

(c) Cylinder $V = \pi r^2 h$

(d) Cone $V = \frac{1}{3}\pi r^2 h$