

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

a b c d e

You have two hours to do this exam. Please write your name and section number on this page.

GOOD LUCK!

- | | |
|--|--|
| 3. <input type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input checked="" type="radio"/> e | 12. <input checked="" type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input type="radio"/> e |
| 4. <input type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input checked="" type="radio"/> d <input type="radio"/> e | 13. <input type="radio"/> a <input type="radio"/> b <input checked="" type="radio"/> c <input type="radio"/> d <input type="radio"/> e |
| 5. <input type="radio"/> a <input type="radio"/> b <input checked="" type="radio"/> c <input type="radio"/> d <input type="radio"/> e | 14. <input checked="" type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input type="radio"/> e |
| 6. <input type="radio"/> a <input checked="" type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input type="radio"/> e | 15. <input type="radio"/> a <input type="radio"/> b <input checked="" type="radio"/> c <input type="radio"/> d <input type="radio"/> e |
| 7. <input checked="" type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input type="radio"/> e | 16. <input type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input checked="" type="radio"/> d <input type="radio"/> e |
| 8. <input type="radio"/> a <input checked="" type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input type="radio"/> e | 17. <input checked="" type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input type="radio"/> e |
| 9. <input type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input checked="" type="radio"/> d <input type="radio"/> e | 18. <input type="radio"/> a <input checked="" type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input type="radio"/> e |
| 10. <input type="radio"/> a <input checked="" type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input type="radio"/> e | 19. <input type="radio"/> a <input type="radio"/> b <input checked="" type="radio"/> c <input type="radio"/> d <input type="radio"/> e |
| 11. <input type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input checked="" type="radio"/> e | 20. <input type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input checked="" type="radio"/> e |

For grading use:

| Multiple Choice | Short Answer |
|--------------------------------|--------------------|
| (number right) (5 points each) | (out of 10 points) |

| | |
|-------|---------------------|
| Total | |
| | (out of 100 points) |

Spring 2019 Exam 2 Short Answer Questions

Write answers on this page. Your work must be clear and legible to be sure you will get full credit.

1. Let $H(x) = e^{g(x)} f(3x^2 + 10)$. Find the derivative, $H'(x)$. **DO NOT SIMPLIFY** your answer. Clearly **circle** your final answer.

Need to use product rule and chain rule

Product rule $H'(x) = (e^{g(x)})' f(3x^2 + 10) + e^{g(x)} \cdot (f(3x^2 + 10))'$

$$= \underbrace{e^{g(x)} \cdot g'(x)}_{\text{chain rule}} f(3x^2 + 10) + e^{g(x)} \underbrace{f'(3x^2 + 10) \cdot (6x)}_{\text{chain rule}}$$

2. The cost function and revenue function (in dollars) for the production and sale of x espresso machines are given as $C(x) = 46000 + 50x$ and $R(x) = 285x - \frac{x^2}{80}$.

Find and simplify the **profit function** and the **marginal profit function**. Circle both of your final answers.

$$P(x) = R(x) - C(x) = 285x - \frac{x^2}{80} - (46000 + 50x)$$

profit function \rightarrow $P(x) = -\frac{x^2}{80} + 235x - 46000$

marginal profit function \rightarrow $P'(x) = -\frac{2x}{80} + 235$

$$P'(x) = \frac{-x}{40} + 235$$

$$f'(x) = 21x^2 + 16x + 9$$

Name: _____

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer on the cover page on this exam.

3. For the function $f(x) = 7x^3 + 8x^2 + 9x + 5$, find the equation of the tangent line to the graph of f at $x = 2$.

Possibilities:

- (a) $y = 111x - 97$
 (b) $y = 125x + 111$
 (c) $y = 111$
 (d) $y = x^3 + 17$
 (e) $y = 125x - 139$

Point-slope form of line is $y - y_1 = m(x - x_1)$

$$f(2) = 7(2)^3 + 8(2)^2 + 9(2) + 5 = 56 + 32 + 18 + 5 = 111$$

$$f'(2) = 21(2)^2 + 16(2) + 9 = 84 + 32 + 9 = 125$$

$$y - 111 = 125(x - 2)$$

$$y - 111 = 125x - 250$$

$$y = 125x - 139$$

4. Find the derivative, $f'(x)$, if $f(x) = \sqrt[5]{6x^3 + 8x^2 + 9x + 7}$.

Possibilities:

- (a) $(1/5)(6x^3 + 8x^2 + 9x + 7)^{-1/5}$
 (b) $(1/5)(18x^2 + 16x + 9)^{-4/5}$
 (c) $(1/5)(6x^3 + 8x^2 + 9x + 7)(18x^2 + 16x + 9)$
 (d) $(1/5)(6x^3 + 8x^2 + 9x + 7)^{-4/5}(18x^2 + 16x + 9)$
 (e) $\sqrt[5]{18x^2 + 16x + 9}$

$$= (6x^3 + 8x^2 + 9x + 7)^{1/5}$$

By the chain rule

$$f'(x) = \frac{1}{5}(6x^3 + 8x^2 + 9x + 7)^{-4/5}(18x^2 + 16x + 9)$$

5. Find the derivative, $f'(x)$, if $f(x) = 8e^{18x} + 17x^e$.

Possibilities:

- (a) $8e^{18x} + 17x^e$
 (b) $\frac{8}{18} \ln(18x) + 17ex^{e-1}$
 (c) $144e^{18x} + 17ex^{e-1}$
 (d) $8 \ln(18x) + 17ex^{e-1}$
 (e) $144xe^{18x-1} + 17ex^{e-1}$

↑ chain rule
 ↑ power rule

$$f'(x) = 8e^{18x}(18) + 17ex^{e-1}$$

$$= 144e^{18x} + 17ex^{e-1}$$

6. Suppose $F(x) = e^x g(19x + 18)$. Find $F'(0)$, given that $g(0) = 9$, $g'(0) = 20$, $g(18) = 17$, $g'(18) = 16$.

Need to use product and chain rules

Possibilities:

- (a) 16
- (b) 321**
- (c) 339
- (d) 19
- (e) 389

$$F'(x) = (e^x)' g(19x+18) + e^x (g(19x+18))'$$

$$= e^x g(19x+18) + e^x g'(19x+18) (19)$$

$$\Rightarrow F'(0) = e^0 g(19(0)+18) + e^0 g'(19(0)+18) \cdot 19$$

$$= g(18) + 19g'(18) = 17 + 19(16) = 321$$

7. Suppose $g(7) = 6$ and $g'(7) = 8$. Find $F'(7)$ if

Need to use quotient rule

$$F(x) = \frac{x^2}{g(x)}$$

Possibilities:

- (a) $-\frac{77}{9}$
- (b) $\frac{77}{9}$**
- (c) $-\frac{44}{7}$
- (d) $-\frac{154}{3}$
- (e) $\frac{4}{3}$

$$F'(x) = \frac{(x^2)' g(x) - x^2 g'(x)}{(g(x))^2}$$

$$= \frac{2xg(x) - x^2 g'(x)}{(g(x))^2}$$

$$\Rightarrow F'(7) = \frac{2(7)g(7) - 7^2 g'(7)}{(g(7))^2} = \frac{14 \cdot 6 - 49 \cdot 8}{36} = -\frac{77}{9}$$

calculator

8. Suppose $H(x) = \sqrt{f(x) + g(x)}$. If $f(9) = 7$, $f'(9) = 8$, $g(9) = 42$, and $g'(9) = 6$, find $H'(9)$.

Possibilities:

Need to use chain rule

- (a) 343
- (b) 1**
- (c) $\frac{1}{28}\sqrt{14}$
- (d) $\sqrt{14}$
- (e) $\frac{1}{14}$

$$H(x) = (f(x) + g(x))^{1/2}$$

$$\rightarrow H'(x) = \frac{1}{2} (f(x) + g(x))^{-1/2} (f'(x) + g'(x))$$

$$\Rightarrow H'(9) = \frac{1}{2} (f(9) + g(9))^{-1/2} (f'(9) + g'(9))$$

$$= \frac{1}{2} (7 + 42)^{-1/2} (8 + 6) = \frac{1}{2\sqrt{49}} \cdot 14 = \frac{14}{14} = 1$$

9. Suppose $F(x) = \ln(g(x))$. If $g(2) = 11$, $g'(2) = 19$, and $g''(2) = 7$, then find $F'(2)$.

Possibilities:

(a) $11/\ln(19)$

(b) $\ln(11)/19$

(c) $\ln(7)$

(d) $19/11$

(e) $11/19$

Need to use chain rule

$$F'(x) = \frac{1}{g(x)} \cdot g'(x)$$

$$\Rightarrow F'(2) = \frac{g'(2)}{g(2)} = \frac{19}{11}$$

10. For the function $f(x) = \begin{cases} x^2 - 9 & x < 3 \\ x^3 - 4 & 3 \leq x < 7 \\ x^{-2} & 7 \leq x \end{cases}$, find the slope of the tangent line to the graph of f at $x = 15$.

Possibilities:

(a) 216

(b) $-\frac{2}{3375}$

(c) 675

(d) $\frac{1}{225}$

(e) 30

$$(x^{-2})' = -2x^{-3}$$

$$\Rightarrow f'(15) = -2(15)^{-3} = \frac{-2}{15^3} = \frac{-2}{3375}$$

x=15 is in this region

11. Find the derivative, $f'(x)$, if $f(x) = \ln(\ln(7 + 9x))$.

Possibilities:

(a) $\frac{1}{\ln(\ln(7 + 9x))} \cdot \frac{1}{\ln(7 + 9x)} \cdot \frac{9}{7 + 9x}$

(b) $\frac{1}{7 + 9x}$

(c) $\left(\frac{9}{7 + 9x}\right) e^{\ln(7 + 9x)}$

(d) $e^{\frac{9}{7 + 9x}}$

(e) $\frac{1}{\ln(7 + 9x)} \cdot \frac{9}{7 + 9x}$

Need to use the chain rule twice

$$f'(x) = \frac{1}{\ln(7 + 9x)} \cdot \frac{1}{7 + 9x} \cdot 9$$

$$= \frac{1}{\ln(7 + 9x)} \cdot \frac{9}{7 + 9x}$$

12. If $f(x) = x^7 + 2x^6 + 9x$ then find the third derivative $f'''(x)$:

Possibilities:

- (a) $210x^4 + 240x^3$
- (b) $210x^4 + 240x^3 + 12x$
- (c) $\frac{7x^6 + 12x^5 + 9}{x^2}$
- (d) $343x^7 + 432x^6$
- (e) $42x^5 + 60x^4$

$$f'(x) = 7x^6 + 12x^5 + 9$$

$$f''(x) = 42x^5 + 60x^4$$

$$f'''(x) = 210x^4 + 240x^3$$

13. If $f(x) = (17x + 38)^{27}$ then $f''(x) =$

Possibilities:

- (a) $27^2 (17)^{27} (17x + 38)$
- (b) $27(26)17^{25}$
- (c) $27(26)(17x + 38)^{25} (17)^2$
- (d) 0
- (e) $27(17x + 38)^{26}$

Need to use chain rule
each time you take
a derivative

$$f'(x) = 27(17x + 38)^{26} (17)$$

$$f''(x) = 27(26)(17x + 38)^{25} (17)(17)$$

14. Find the derivative, $f'(x)$, of $f(x) = \frac{1}{x^{60}} = x^{-60}$

Possibilities:

- (a) $-60x^{-61}$
- (b) $-60x^{-59}$
- (c) $60x^{59}$
- (d) $1/(60x^{59})$
- (e) $1/(60x^{61})$

$$\Rightarrow f'(x) = -60x^{-61}$$

-
15. If $\$7000$ dollars is invested at 6% annual interest compounded continuously, what is the value of the investment at the end of 3 years?

Possibilities:

- (a) \$5846.89
- (b) \$8260.00
- (c) \$8380.52
- (d) \$12600.00
- (e) \$42347.53

P_0 $\rightarrow r = .06$

$$P(t) = P_0 e^{rt}$$
$$P(3) = 7000 e^{(.06)(3)}$$
$$\approx 8380.52$$

-
16. A bacteria culture starts with 8000 bacteria and triples after 13 hours. If we express the number of bacteria after t hours as $y(t) = a \cdot e^{kt}$, find the value of k .

Possibilities:

- (a) $8000 / \ln(3)$
- (b) $\ln(3) / \ln(13)$
- (c) 8000
- (d) $\ln(3) / 13$
- (e) $13 / \ln(3)$

When $t=13$, there are 24000 bacteria

$$\Rightarrow 24000 = 8000 e^{k(13)}$$
$$\Rightarrow \ln(3) = \ln(e^{13k})$$
$$\Rightarrow \ln(3) = 13k$$
$$\Rightarrow \frac{\ln(3)}{13} = k$$

17. A drug is injected into the bloodstream of a patient. The concentration of the drug in the bloodstream (in milligrams per cubic centimeter) t hours after the injection is given by

$$C(t) = \frac{.21t}{t^2 + 7}$$

Find the instantaneous rate of change of the drug concentration with respect to time at $t = 1$ hour.

Possibilities:

- (a) 0.020 units per hour
 (b) 0.026 units per hour
 (c) 0.105 units per hour
 (d) 6.000 units per hour
 (e) 33.333 units per hour

Find $C'(t)$ and plug in $t=1$

$$C'(t) = \frac{(.21t)'(t^2+7) - .21t(t^2+7)'}{(t^2+7)^2}$$

$$= \frac{.21(t^2+7) - .21t(2t)}{(t^2+7)^2}$$

$$\rightarrow C'(1) = \frac{.21(1^2+7) - .21(1)(2(1))}{(1^2+7)^2} = \frac{.21(8) - .21(2)}{64}$$

$$= .020$$

18. The price-demand function for the production of x microwaves is given as

$$p = 230 - \frac{x}{60}$$

Evaluate the **marginal revenue** function at $x = 1000$.

Possibilities:

- (a) \$ - 16.67
 (b) \$196.67
 (c) \$213333.33
 (d) \$213.33
 (e) \$ - 1770.00

$$R(x) = p \cdot x = \left(230 - \frac{x}{60}\right) \cdot x$$

$$= 230x - \frac{x^2}{60}$$

$$\Rightarrow R'(x) = 230 - \frac{x}{30}$$

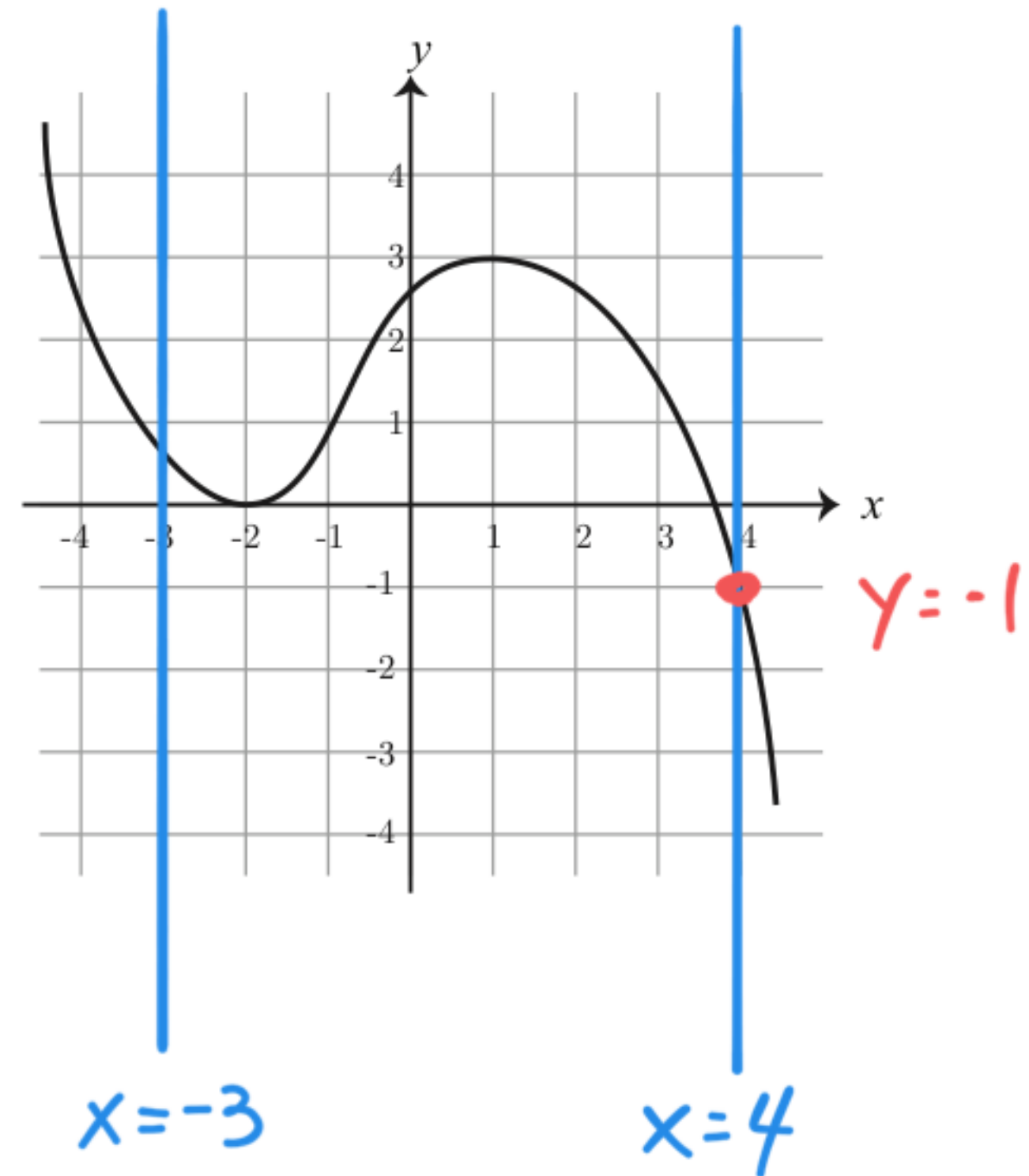
$$\Rightarrow R'(1000) = 230 - \frac{1000}{30} \approx 196.67$$

19. The graph of $y = f(x)$ is shown below. What is the minimum value of $f(x)$ on the interval $[-3, 4]$?

Possibilities:

- (a) 4
- (b) 3
- (c) -1
- (d) -2
- (e) 1

Smallest y value
on the interval
 $[-3, 4]$ is -1



20. Find the minimum value of $g(x) = x^3 + 9x^2 + 170$ on the interval $[-1, 5]$.

Possibilities:

- (a) 164
- (b) 278
- (c) 178
- (d) 520
- (e) 170

① Set $g'(x) = 0$ and solve for x

$$g'(x) = 3x^2 + 18x = 0$$

$$3x(x+6) = 0$$

$$x = 0, x = -6$$

Not in $[-1, 5]$

② Plug $x=0$
and endpoints into $g(x)$

$$g(-1) = (-1)^3 + 9(-1)^2 + 170 = 178$$

$$g(0) = 0^3 + 9(0)^2 + 170 = 170 \text{ min}$$

$$g(5) = 5^3 + 9(5)^2 + 170 = 125 + 225 + 170 = 520$$

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

a b c d e

You have two hours to do this exam. Please write your name and section number on this page.

GOOD LUCK!

- | | |
|--|--|
| 3. <input type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input checked="" type="radio"/> e | 12. <input checked="" type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input type="radio"/> e |
| 4. <input type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input checked="" type="radio"/> d <input type="radio"/> e | 13. <input type="radio"/> a <input type="radio"/> b <input checked="" type="radio"/> c <input type="radio"/> d <input type="radio"/> e |
| 5. <input type="radio"/> a <input type="radio"/> b <input checked="" type="radio"/> c <input type="radio"/> d <input type="radio"/> e | 14. <input checked="" type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input type="radio"/> e |
| 6. <input type="radio"/> a <input checked="" type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input type="radio"/> e | 15. <input type="radio"/> a <input type="radio"/> b <input checked="" type="radio"/> c <input type="radio"/> d <input type="radio"/> e |
| 7. <input checked="" type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input type="radio"/> e | 16. <input type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input checked="" type="radio"/> d <input type="radio"/> e |
| 8. <input type="radio"/> a <input checked="" type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input type="radio"/> e | 17. <input checked="" type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input type="radio"/> e |
| 9. <input type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input checked="" type="radio"/> d <input type="radio"/> e | 18. <input type="radio"/> a <input checked="" type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input type="radio"/> e |
| 10. <input type="radio"/> a <input checked="" type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input type="radio"/> e | 19. <input type="radio"/> a <input type="radio"/> b <input checked="" type="radio"/> c <input type="radio"/> d <input type="radio"/> e |
| 11. <input type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input checked="" type="radio"/> e | 20. <input type="radio"/> a <input type="radio"/> b <input type="radio"/> c <input type="radio"/> d <input checked="" type="radio"/> e |

For grading use:

| | |
|-----------------|--------------------|
| Multiple Choice | Short Answer |
| (number right) | (5 points each) |
| | (out of 10 points) |

| | |
|-------|---------------------|
| Total | |
| | (out of 100 points) |