MA123 — Elem. Calculus Fall 2016
Exam 3 2016-11-17

Name: SOLUTIONS Sec.: _____

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name on this page, and at the top of page three.

GOOD LUCK!

3. (a) (b) (c) (d) (e)
4. (a) (b) (c) (d) (e)
5. (a) (b) (c) (d) (e)
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17. (a) (b) (c) (d) (e)
18. (a) (b) (c) (d) (e)
19. (a) (b) (c) (d) (e)
20. (a) (b) (c) (d) (e)

For grading use:

Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

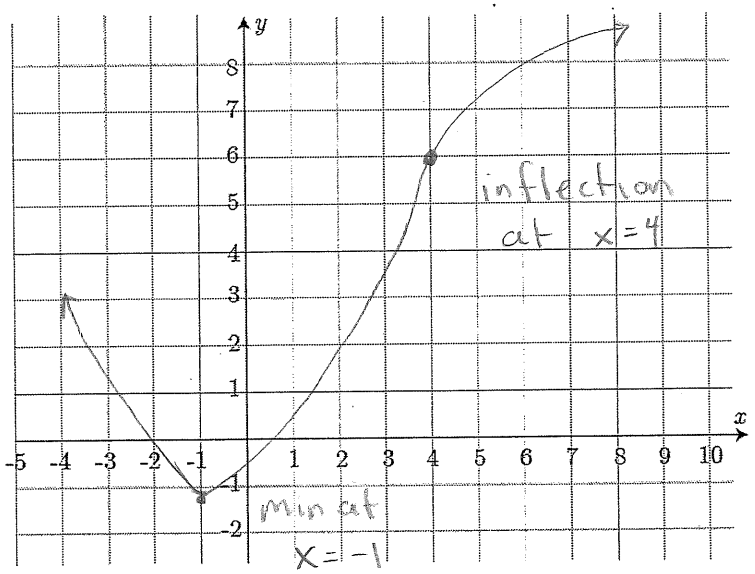
Total	
	(out of 100 points)

Name: **Last 4 digits of Student ID:**

Fall 2016 Exam 3 Short Answer Questions

Write answers on this page. Your work must be clear and legible to be sure you will get full credit.

1. Sketch the graph of a **continuous** function $y=f(x)$ which satisfies $f'(x)<0$ for $x<-1$; $f'(x)>0$ for $x>-1$; f is concave up for $x<4$; concave down for $x>4$.



2. Suppose the product of x and y is 37 and both x and y are positive. What is the minimum possible sum of x and y? *You must **clearly use calculus** to find **and justify** your answer. Your final answer does **not** need to be simplified.*

$S = x+y$

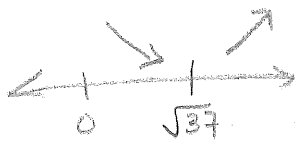
$xy = 37 \Rightarrow y = \frac{37}{x}$

$\Rightarrow S = x + \frac{37}{x} = x + 37x^{-1}$ interval $(0, \infty)$

$S' = 1 - 37x^{-2} = 1 - \frac{37}{x^2}$

$S' = 0$ when $1 - \frac{37}{x^2} = 0$ $\quad 1 = \frac{37}{x^2}$ $\quad x^2 = 37$ $\quad x = \sqrt{37}$

test



min at $x = \sqrt{37}$

$S = \sqrt{37} + \frac{37}{\sqrt{37}} = \sqrt{37} + \sqrt{37}$

Minimum possible sum: $2\sqrt{37} \approx 12.166$

Name: SOLUTIONS

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

3. Where is the function $f(t) = \dfrac{1}{t-60}$ decreasing?

Possibilities:

(a) $f(t)$ is always decreasing except at t = 60
(b) $-1 < t < 60$
(c) $t > 60$
(d) $f(t)$ is never decreasing
(e) $t < 60$

$f(t) = (t-60)^{-1}$

$f'(t) = -(t-60)^{-2} = \dfrac{-1}{(t-60)^2}$

$f'(t)$ is negative except at $t=60$ where it is undefined.

test

$\Rightarrow$ decreasing on $(-\infty, 60) \cup (60, \infty)$

4. Where is the function $f(t) = t^4 - 12t^3 - 1$ concave up?

Possibilities:

(a) $t > 9$
(b) $f(t)$ is always concave up
(c) $t < 9$
(d) $t < 0$ and $t > 6$
(e) $0 < t < 6$

$f(t)$ is concave up when $f''(t) > 0$

$f'(t) = 4t^3 - 36t^2$

$f''(t) = 12t^2 - 72t$

$f''(t) = 0$ when $12t(t-6) = 0 \Rightarrow t = 0, \; t = 6$

test $f''(t)$

$\Rightarrow$ concave up on $(-\infty, 0) \cup (6, \infty)$

5. Suppose the derivative of $g(t)$ is $g'(t) = 8(t-2)(t-6)(t-4)$. For t in which interval(s) is g increasing?

Possibilities:

(a) $(-\infty, 4 - \frac{2}{3}\sqrt{3}) \cup (4 + \frac{2}{3}\sqrt{3}, \infty)$

(b) $(-\infty, 2) \cup (4, 6)$

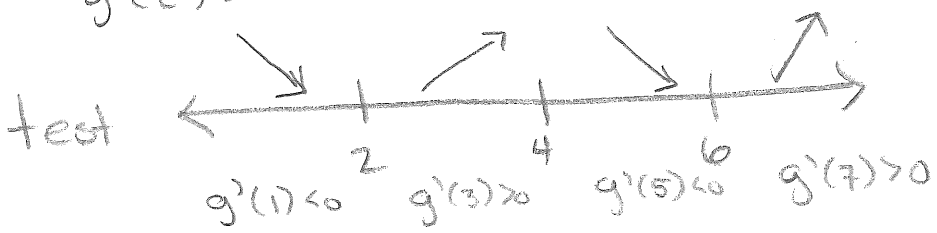
(c) $(4 - \frac{2}{3}\sqrt{3}, 4 + \frac{2}{3}\sqrt{3})$

(d) $(2, 4) \cup (6, \infty)$

(e) $(2, 4) \cup (6, 8)$

$g(t)$ is increasing when $g'(t) > 0$

$g'(t) = 0$ when $t = 2, 4, 6$



$\Rightarrow$ g is increasing on $(2,4) \cup (6,\infty)$

6. Suppose the derivative of $g(t)$ is $g'(t) = 14(t-2)(t-8)$. For t in which interval(s) is g concave up?

Possibilities:

(a) $(-\infty, 2) \cup (8, \infty)$

(b) $(-\infty, 5)$

(c) $(2, 8)$

(d) $(2, 5) \cup (8, 14)$

(e) $(5, \infty)$

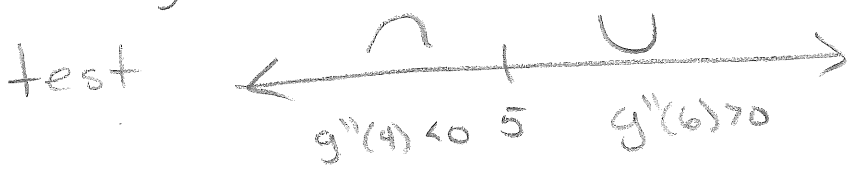
g is concave up when $g''(t) > 0$

by the product rule,

$g''(t) = 14(1 \cdot (t-8) + (t-2) \cdot 1)$

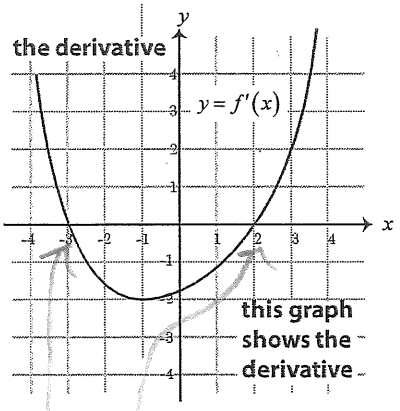
$= 14(2t - 10)$

$g''(t) = 0$ when $2t - 10 = 0 \Rightarrow t = 5$



concave up on $(5, \infty)$

7. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$.
Where is the original function $f(x)$ decreasing?



Possibilities:

(a) $(-\infty, -1)$

(b) $(-3, 2)$

(c) $(-1, \infty)$

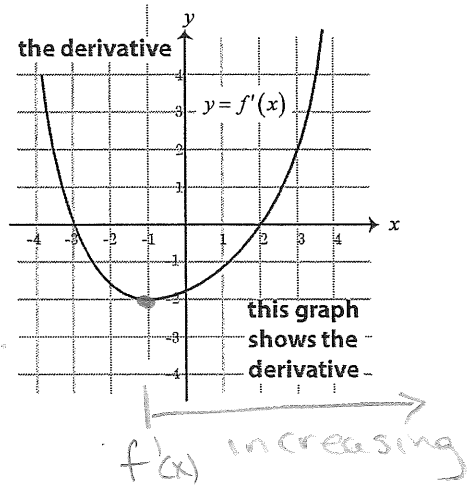
(d) $(-2, \infty)$

(e) $(-\infty, -3)$ and $(2, \infty)$

$f(x)$ is decreasing when $f'(x) < 0$. This happens between $x = -3$ and $x = 2$

$\Rightarrow$ f is decreasing on $(-3, 2)$

8. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$.
Where is the original function $f(x)$ concave up?



Possibilities:

(a) $(-1, \infty)$

(b) $(-\infty, -3)$ and $(2, \infty)$

(c) $(-3, 2)$

(d) $(-2, \infty)$

(e) $(-\infty, -1)$

$f(x)$ is concave up when $f'(x)$ is increasing. This happens when $x > -1$

$\Rightarrow$ f is concave up on $(-1, \infty)$

9. Find the area of the largest rectangle whose sides are parallel to the coordinate axes, whose bottom-left corner is at $(0, 0)$ and whose top-right corner is on the graph of $y = 21x - x^2$.

Possibilities:

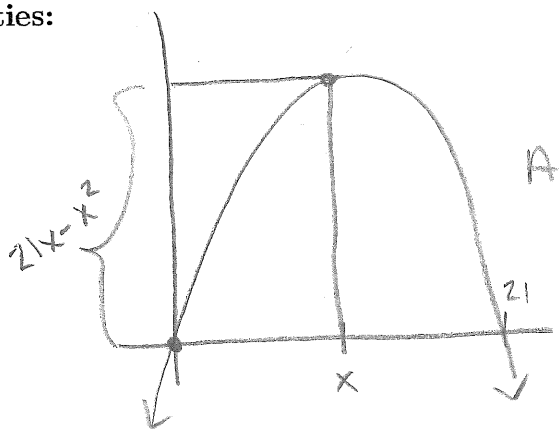
(a) $\frac{9261}{8}$

(b) 1372

(c) 0

(d) $\frac{21}{2}$

(e) 420



Interval $[0, 21]$

$A(x)$ = Area of rectangle $= b \cdot h$

$= x \cdot (21x - x^2)$

$= 21x^2 - x^3$

$A'(x) = 42x - 3x^2 = 3x(14 - x)$

$\Rightarrow$ critical values are $x = 0, 14$

check $A(0), A(14), A(21)$

$A(0) = 0$

$A(14) = 21(14)^2 - 14^3 = 1372$

$A(21) = 0$

10. A box is constructed out of two different types of metal. The metal for the top and bottom, which are both square, costs \$3 per square foot, and the metal for the four sides costs \$11 per square foot. The box has a volume of 30 cubic feet. If we find the dimensions that minimize cost, what is the length of the base?

Possibilities:

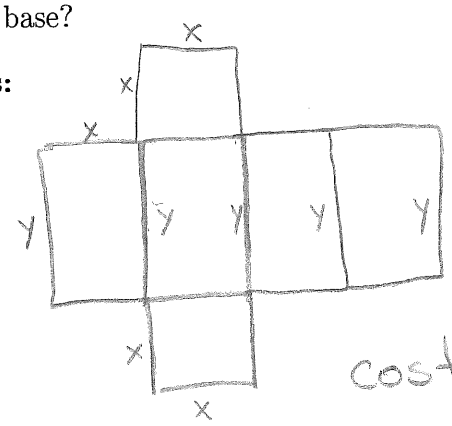
(a) 5.29 feet

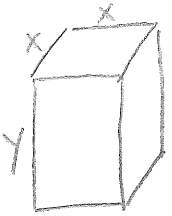
(b) 5.79 feet

(c) 4.29 feet

(d) 4.79 feet

(e) 6.29 feet





$V = x^2 y = 30$

$\Rightarrow y = \frac{30}{x^2}$

$\text{cost} = 3x^2 \cdot 2 + 11xy \cdot 4$

$= 6x^2 + 44xy$

$= 6x^2 + \frac{1320}{x} = 6x^2 + 1320x^{-1}$

$C'(x) = 12x - 1320x^{-2} \stackrel{?}{=} 0$

$12x = \frac{1320}{x^2} \Rightarrow x^3 = 110 \Rightarrow x \approx 4.79$

$C'(1) < 0$ 4.79 $C'(5) > 0$

11. Suppose the derivative of $H(s)$ is given by $H'(s) = 1/(s^2+8)$. Find the value of s in the interval $[-10, 10]$ where $H(s)$ takes on its maximum.

Possibilities:

(a) 10

(b) $-\frac{1}{8}$

(c) 8

(d) -10

(e) -8

Notice that $H'(s) = \frac{1}{s^2+8} > 0$ for all s

$\Rightarrow$ $H(s)$ is always increasing

$\Rightarrow$ $H(s)$ takes on its maximum at the right most value

$\Rightarrow$ $H(s)$ takes on its max at $s=10$

12. Find the critical numbers of the function $f(x) = 2xe^{17x}$.

Possibilities:

(a) $-\frac{2}{17}, 0$

(b) 0

(c) $-\frac{1}{17}$

(d) $-\frac{2}{17}$

(e) $-\frac{1}{17}, 0, e^{17}$

Critical numbers occure when $f'(x) = 0$ or when $f'(x)$ is undefined.

By the product rule,

$f'(x) = 2e^{17x} + 2x(17e^{17x})$

$= 2e^{17x}(1 + 17x)$

never zero

$1 + 17x = 0$

$\Rightarrow x = -\frac{1}{17}$

13. The graph of $y = f(x)$ shown below consists of straight lines. Evaluate the definite integral $\int_{-3}^{3} f(x)\,dx$.

Possibilities:

(a) 17.5

(b) 12

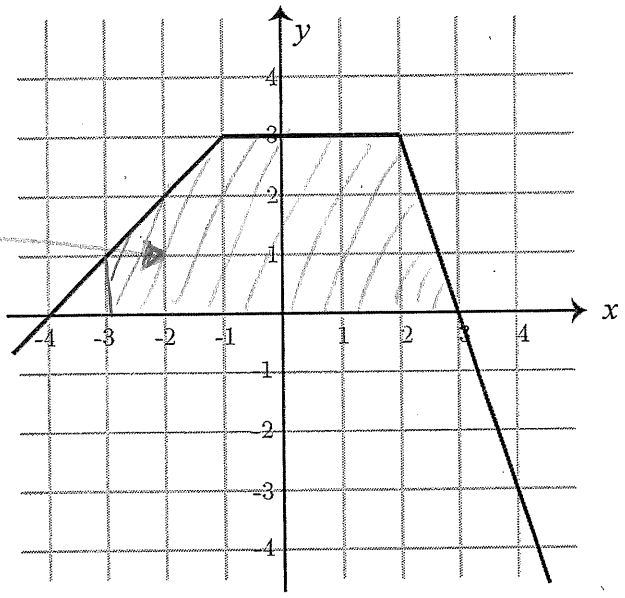
(c) 16

(d) 14.5

(e) 19

$\int_{-3}^{3} f(x)\,dx =$ Area

$= 14.5$



14. The graph of $y = f(x)$ shown below includes a semicircle and a straight line. Evaluate the definite integral $\int_{-4}^{4} f(x)\,dx$. Use $\pi = 3.14$.

Possibilities:

(a) -4.56

(b) 14.28

(c) -14.28

(d) $-.28$

(e) 1.72

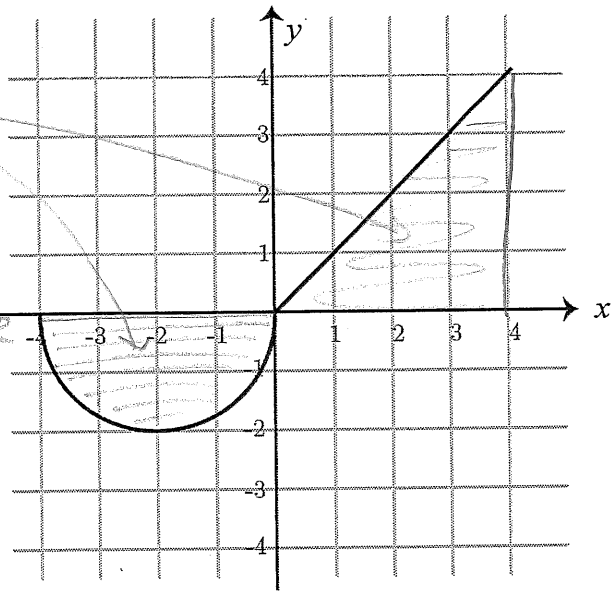
$\int_{-4}^{4} f(x)\,dx =$ signed Area

$=$ Area of triangle $-$ Area of half circle

$= \frac{1}{2} b \cdot h - \frac{\pi r^2}{2}$

$= \frac{1}{2} 4 \cdot 4 - \frac{(3.14)(2)^2}{2}$

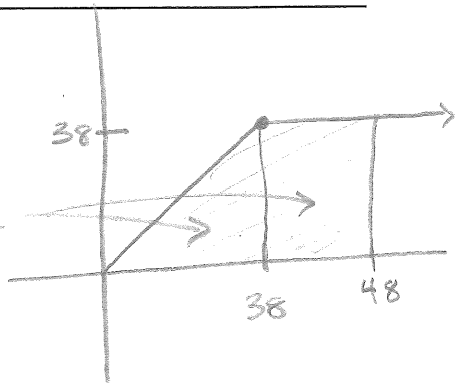
$= 1.72$



15. Given the function $f(x) = \begin{cases} x & \text{if } x < 38 \\ 38 & \text{if } x \geq 38 \end{cases}$

evaluate the definite integral

$$\int_0^{48} f(x)\,dx = \text{Area}$$



Possibilities:

(a) 1102

(b) 1103

(c) 1104

(d) 1105

(e) 1106

$$= \text{Area of triangle} + \text{area of rectangle}$$

$$= \frac{1}{2} b \cdot h + l \cdot w$$

$$= \frac{1}{2}(38)(38) + (10)(38)$$

$$= 722 + 380$$

$$= 1102$$

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16. Suppose that $\int_{10}^{24} f(x)\,dx = 17$ and $\int_1^{24} f(x)\,dx = 8$. Find the value of $\int_1^{10} f(x)\,dx$.

Possibilities:

(a) -25

(b) -9

(c) -1

(d) 25

(e) 9

$$\int_1^{24} f(x)\,dx = \int_1^{10} f(x)\,dx + \int_{10}^{24} f(x)\,dx$$

$$\Rightarrow 8 = \int_1^{10} f(x)\,dx + 17$$

$$\Rightarrow -9 = \int_1^{10} f(x)\,dx$$

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17. Suppose that $\int_2^{24} f(x)\,dx = 11$. Find the value of $\int_2^{24} (3f(x)+2)\,dx$.

Possibilities:

(a) 81

(b) 35

(c) 55

(d) 77

(e) 39

$$\int_2^{24} (3f(x)+2)\,dx$$

$$= 3\int_2^{24} f(x)\,dx + \int_2^{24} 2\,dx \quad \leftarrow \text{Area of rectangle with base} = 22, \text{height} = 2$$

$$= 3(11) + (24-2)(2)$$

$$= 33 + 44$$

$$= 77$$

18. Find the average value of $f(x)$ on the interval $[5, 13]$ given that $f(x) = \begin{cases} 70 & \text{if } x < 8 \\ -10 & \text{if } x \geq 8. \end{cases}$

Possibilities:

(a) -10

(b) 6

(c) 80

(d) 30

(e) 20

$$\text{Average value} = \frac{1}{13-5}\int_5^{13} f(x)\,dx$$

$$= \frac{1}{8}\left(\int_5^8 70\,dx + \int_8^{13} (-10)\,dx\right)$$

$$= \frac{1}{8}\left(70 \cdot 3 + (-10)(5)\right)$$

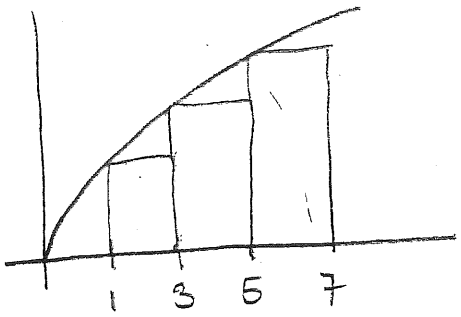
$$= \frac{1}{8}(210 - 50) = 20$$

f(x)

19. Estimate the area under the graph of $y = -x^2 + 30x$ for x between 1 and 7, by using a partition that consists of 3 equal subintervals of $[1, 7]$ and use the left endpoint of each subinterval as a sample point.

Possibilities:

(a) 792
(b) 606
(c) 470
(d) 734
(e) 367



3 equal subintervals
$\Rightarrow$ base of each rectangle = 2

Area of rectangles $= 2 \cdot f(1) + 2f(3) + 2f(5)$
$= 2 \cdot (-1^2 + 30(1)) + 2(-3^2 + 30(3)) + 2(-5^2 + 30(5))$
$= 2(29) + 2(81) + 2(125)$
$= 470$

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20. Suppose you estimate the area under the graph of $f(x) = x^3$ from $x = 6$ to $x = 46$ by adding the areas of the rectangles as follows: partition the interval into 20 equal subintervals and use the right endpoint of each interval to determine the height of the rectangle. What is the area of the 9th rectangle?

Possibilities:

(a) 13824
(b) 24380
(c) 27648
(d) 21296
(e) 1218240

20 subintervals $\Rightarrow$ base of each rectangle is 2.

Right endpoint rule $\Rightarrow$ 1st rectangle has area $(2) f(8)$
$= 2 \cdot 8^3 = 1024$

Area of 2nd rectangle $= 2 \cdot 10^3 = 2000$

Notice that area of n^{th} rectangle is $2(6 + 2n)^3$

$\Rightarrow$ Area of 9^{th} rectangle is $2(6 + 2(9))^3$
$= 2(24)^3 = 27648$

Some Formulas

1. **Areas:**
   (a) Triangle $A = \frac{bh}{2}$
   (b) Circle $A = \pi r^2$
   (c) Rectangle $A = lw$
   (d) Trapezoid $A = \frac{h_1 + h_2}{2} b$

2. **Volumes:**
   (a) Rectangular Solid $V = lwh$
   (b) Sphere $V = \frac{4}{3}\pi r^3$
   (c) Cylinder $V = \pi r^2 h$
   (d) Cone $V = \frac{1}{3}\pi r^2 h$