

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

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a    b    c    d    e

You have two hours to do this exam. Please write your name and section number on this page.

**GOOD LUCK!**

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For grading use:

<b>Multiple Choice</b>	<b>Short Answer</b>
(number right)   (5 points each)	(out of 10 points)

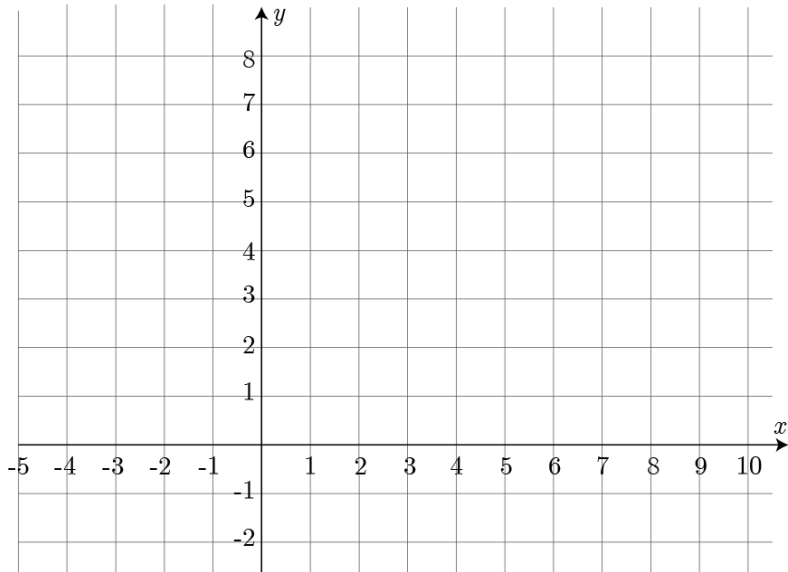
<b>Total</b>	
	(out of 100 points)

### Spring 2018 Exam 3 Short Answer Questions

Write answers on this page. Your work must be clear and legible to be sure you will get full credit.

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1. Sketch the graph of a **continuous** function  $y = f(x)$  which satisfies  $f'(x) < 0$  for  $x < 7$ ;  $f'(x) > 0$  for  $x > 7$ ;  $f''(x) < 0$  for  $x < 2$ ;  $f''(x) > 0$  for  $x > 2$ .



2. The product of two positive real numbers  $x$  and  $y$  is 21. Find the minimum value of the expression  $3x + 2y$ . You must **clearly use calculus** to find **and justify** your answer. Your final answer does **not** need to be simplified.

Minimum possible value: \_\_\_\_\_

**Multiple Choice Questions**

*Show all your work on the page where the question appears.  
Clearly mark your answer both on the cover page on this exam  
and in the corresponding questions that follow.*

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3. Where is the function  $f(t) = \frac{1}{t - 23}$  decreasing?

**Possibilities:**

- (a)  $t > 23$
- (b)  $-1 < t < 23$
- (c)  $f(t)$  is always decreasing except at  $t = 23$
- (d)  $f(t)$  is never decreasing
- (e)  $t < 23$

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4. Where is the function  $f(t) = t^4 - 12t^3 - 8$  concave up?

**Possibilities:**

- (a)  $0 < t < 6$
  - (b)  $t < 0$  and  $t > 6$
  - (c)  $t < 9$
  - (d)  $f(t)$  is always concave up
  - (e)  $t > 9$
-

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5. Suppose the derivative of  $g(t)$  is  $g'(t) = 11(t - 3)(t - 9)^2$ . Find all interval(s) of values of  $t$  in which  $g$  is **increasing**.

**Possibilities:**

- (a)  $(3, 11)$
- (b)  $(-\infty, 3) \cup (9, \infty)$
- (c)  $(-\infty, 3)$
- (d)  $(3, \infty)$
- (e)  $(-\infty, 9)$

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6. Suppose the derivative of  $g(t)$  is  $g'(t) = 11t^2 - 132t + 297$ . For  $t$  in which interval(s) is  $g$  concave up?

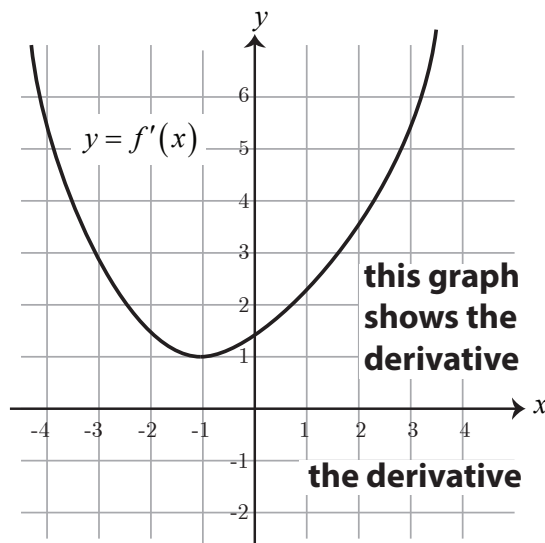
**Possibilities:**

- (a)  $(6, \infty)$
- (b)  $(-\infty, 3) \cup (9, \infty)$
- (c)  $(-\infty, 6)$
- (d)  $(3, 6) \cup (9, 11)$
- (e)  $(3, 9)$

- 
7. The following is the graph of the **derivative**,  $f'(x)$ , of the function  $f(x)$ .  
Where is the original function  $f(x)$  decreasing?

**Possibilities:**

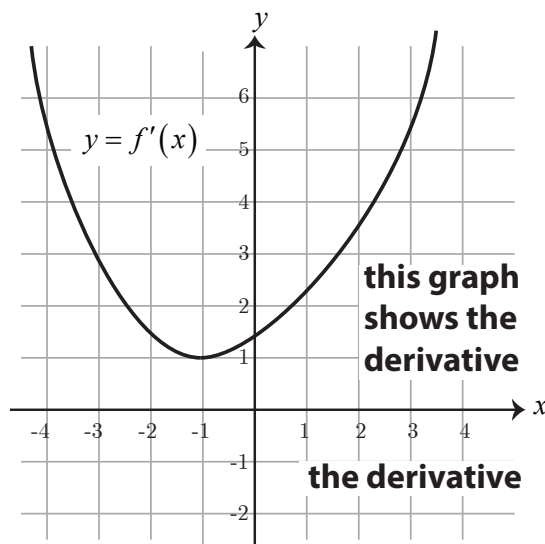
- (a)  $(1, \infty)$
- (b)  $(-\infty, -1)$
- (c) nowhere
- (d)  $(-1, \infty)$
- (e)  $(-\infty, \infty)$



- 
8. The following is the graph of the **derivative**,  $f'(x)$ , of the function  $f(x)$ .  
Where is the original function  $f(x)$  concave up?

**Possibilities:**

- (a)  $(1, \infty)$
- (b)  $(-\infty, -1)$
- (c) nowhere
- (d)  $(-\infty, \infty)$
- (e)  $(-1, \infty)$



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9. Find the critical numbers of the function

$$f(x) = \frac{8x}{3x^2 + 12}.$$

**Possibilities:**

- (a)  $-4, 0$
- (b)  $-2, 2$
- (c)  $-\frac{8}{3}, \frac{8}{12}$
- (d)  $-4, 8$
- (e)  $-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}$

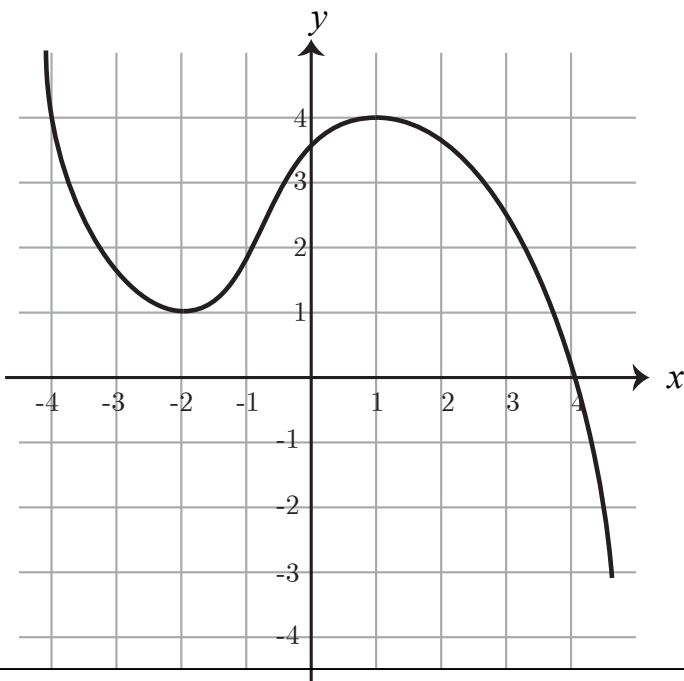
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10. Consider the graph of the original function,  $f(x)$ .

For this function, what are the signs of  $f'(-3)$  and  $f''(-3)$ ?

**Possibilities:**

- (a)  $f'(-3) < 0$  and  $f''(-3) > 0$
- (b)  $f'(-3) > 0$  and  $f''(-3) > 0$
- (c)  $f'(-3) > 0$  and  $f''(-3) < 0$
- (d)  $f'(-3) = 0$  and  $f''(-3) < 0$
- (e)  $f'(-3) < 0$  and  $f''(-3) < 0$



- 
11. Find the area of the largest rectangle whose sides are parallel to the coordinate axes, whose bottom-left corner is at  $(0,0)$  and whose top-right corner is on the graph of  $y = 21x - x^2$ .

**Possibilities:**

- (a)  $\frac{21}{2}$
- (b)  $\frac{9261}{8}$
- (c) 0
- (d) 420
- (e) 1372

- 
12. An open box is to be made out of a 6-inch by 18-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. If we find the dimensions of the resulting box that has the largest volume, what is its height?

**Possibilities:**

- (a) 1.15 inches
- (b) 1.25 inches
- (c) 1.35 inches
- (d) 1.45 inches
- (e) 1.55 inches

- 
13. Given the function  $f(x) = \begin{cases} 0 & \text{if } x < -11 \\ 5 & \text{if } -11 \leq x < 0 \\ -50 & \text{if } 0 \leq x < 6 \\ 0 & \text{if } x \geq 6 \end{cases}$

evaluate the definite integral

$$\int_{-11}^4 f(x) \, dx$$

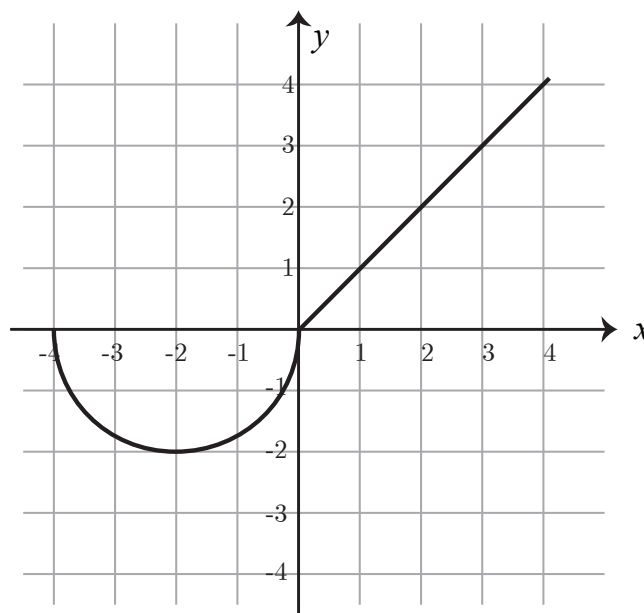
**Possibilities:**

- (a)  $-145$
- (b)  $55$
- (c)  $255$
- (d)  $-245$
- (e)  $-255$

- 
14. The graph of  $y = f(x)$  shown below includes a semicircle and a straight line. Evaluate the definite integral  $\int_{-4}^4 f(x) \, dx$ .

**Possibilities:**

- (a)  $-2\pi + 6$
- (b)  $-4\pi + 8$
- (c)  $-2\pi - 8$
- (d)  $2\pi + 8$
- (e)  $-2\pi + 8$





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15. Suppose that  $\int_2^7 f(x) dx = 24$ ,  $\int_{28}^{35} f(x) dx = 48$ , and  $\int_2^{35} f(x) dx = 11$ . Find the value of  $\int_7^{28} f(x) dx$ .

**Possibilities:**

- (a) 83
- (b) 13
- (c) -83
- (d) -61
- (e) -93

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16. Suppose that  $\int_3^{18} f(x) dx = 8$ . Find the value of  $\int_3^{18} (3f(x) + 60) dx$ .

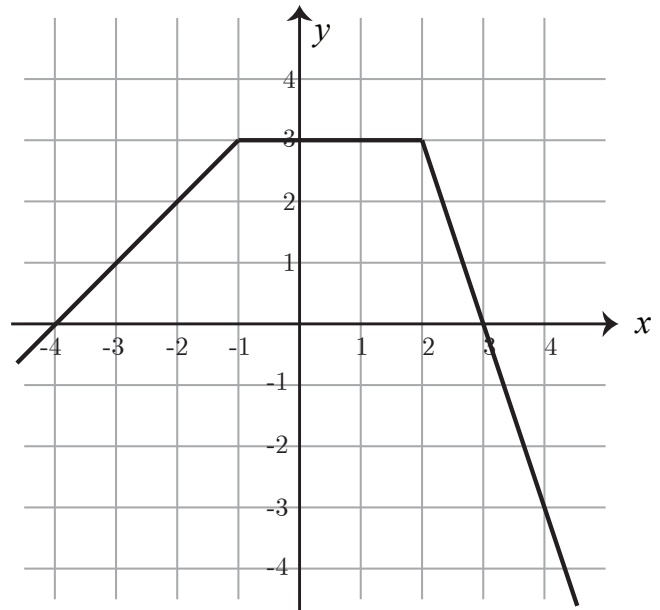
**Possibilities:**

- (a) 84
  - (b) 1104
  - (c) 924
  - (d) 204
  - (e) 39
-

- 
17. The graph of  $y = f(x)$  shown below consists of straight lines. Find the **average value** of  $f(x)$  on the interval  $[-3, 2]$ .

**Possibilities:**

- (a) 2
- (b)  $\frac{13}{10}$
- (c) 3
- (d)  $\frac{13}{5}$
- (e)  $\frac{2}{5}$



- 
18. Estimate the area under the graph of  $y = x^2 + 7$  for  $x$  between 0 and 6, by using a partition that consists of 3 equal subintervals of  $[0, 6]$  and use the left endpoint of each subinterval as a sample point.

**Possibilities:**

- (a) 168
- (b) 82
- (c) 114
- (d) 154
- (e) 77

- 
19. Suppose you estimate the area under the graph of  $f(x) = \frac{1}{x}$  from  $x = 6$  to  $x = 24$  by adding the areas of the rectangles as follows: partition the interval into 6 equal subintervals and use the right endpoint of each interval to determine the height of the rectangle. What is the area of the 3<sup>rd</sup> rectangle?

**Possibilities:**

- (a)  $\frac{341}{280}$
- (b)  $\frac{1}{15}$
- (c)  $\frac{1}{5}$
- (d)  $-2 \ln(2) + \ln(5)$
- (e)  $\frac{1}{4}$

- 
20. The rate (in liters per minute) at which water drains from a tank is recorded at half-minute intervals. Use the average of the left- and right-endpoint approximations to estimate the total amount of water drained during the first 2 minutes.

t min	0	.5	1	1.5	2
l/min	4	8	16	22	23

Use all five measurements in your estimate.

**Possibilities:**

- (a) 29.75 liters
- (b) 11.50 liters
- (c) 25.00 liters
- (d) 8.00 liters
- (e) 36.50 liters

## Some Formulas

### 1. Areas:

(a) Triangle  $A = \frac{bh}{2}$

(b) Circle  $A = \pi r^2$

(c) Rectangle  $A = lw$

(d) Trapezoid  $A = \frac{h_1 + h_2}{2} b$

### 2. Volumes:

(a) Rectangular Solid  $V = lwh$

(b) Sphere  $V = \frac{4}{3}\pi r^3$

(c) Cylinder  $V = \pi r^2 h$

(d) Cone  $V = \frac{1}{3}\pi r^2 h$

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Total	
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