

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

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**GOOD LUCK!**

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For grading use:

Multiple Choice	Short Answer
(number right) (5 points each)	(out of 10 points)

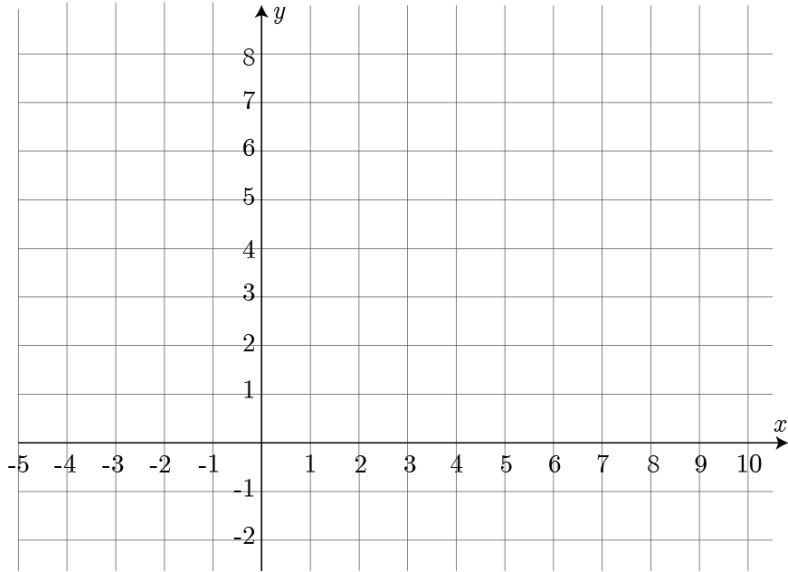
Total	
	(out of 100 points)

**Spring 2019 Exam 3 Short Answer Questions**

Write answers on this page. Your work must be clear and legible to be sure you will get full credit.

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1. Sketch the graph of a **continuous** function  $y = f(x)$  for which  $f$  is increasing on  $(-\infty, 5)$ , decreasing on  $(5, \infty)$ ,  $f''(x) > 0$  on  $(-\infty, 2)$  and  $(7, \infty)$ ;  $f''(x) < 0$  on  $(2, 7)$ .



2. Suppose we know two nonnegative numbers  $x$  and  $y$  satisfying  $4x + y = 13$ . Find the maximum possible value of their product  $xy$ . You must CLEARLY USE CALCULUS to find and justify your answer. Your final answer does **not** need to be simplified.

Maximum possible product: \_\_\_\_\_

Name: \_\_\_\_\_

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**Multiple Choice Questions**

*Show all your work on the page where the question appears.  
Clearly mark your answer both on the cover page on this exam  
and in the corresponding questions that follow.*

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3. Where is the function  $f(t) = t^3 + 3t^2 - 72t + 4$  decreasing?

**Possibilities:**

- (a)  $t < -1$
  - (b)  $t < -6$  and  $t > 4$
  - (c)  $f(t)$  is always decreasing
  - (d)  $t > -1$
  - (e)  $-6 < t < 4$
- 

4. Where is the function  $f(t) = t^4 - 16t^3 - 9$  concave up?

**Possibilities:**

- (a)  $t > 12$
  - (b)  $t < 0$  and  $t > 8$
  - (c)  $0 < t < 8$
  - (d)  $t < 12$
  - (e)  $f(t)$  is always concave up
-

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5. Suppose the derivative of  $g(t)$  is  $g'(t) = 11(t - 2)^2(t - 10)$ . For  $t$  in which interval(s) is  $g$  increasing?

**Possibilities:**

- (a)  $(2, 10)$
- (b)  $(-\infty, 10)$
- (c)  $(2, 10) \cup (11, \infty)$
- (d)  $(10, \infty)$
- (e)  $(-\infty, 2) \cup (10, \infty)$

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6. Suppose the derivative of  $g(t)$  is  $g'(t) = 11t^2 - 132t + 220$ . For  $t$  in which interval(s) is  $g$  concave up?

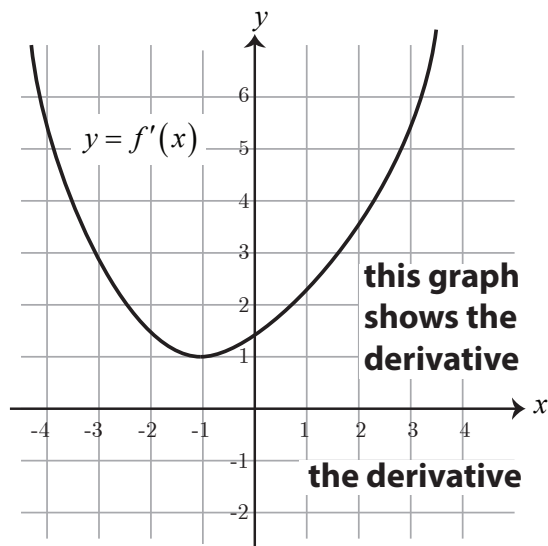
**Possibilities:**

- (a)  $(2, 10)$
  - (b)  $(-\infty, 2) \cup (10, \infty)$
  - (c)  $(2, 6) \cup (10, 11)$
  - (d)  $(-\infty, 6)$
  - (e)  $(6, \infty)$
-

- 
7. The following is the graph of the **derivative**,  $f'(x)$ , of the function  $f(x)$ .  
Where is the original function  $f(x)$  decreasing?

**Possibilities:**

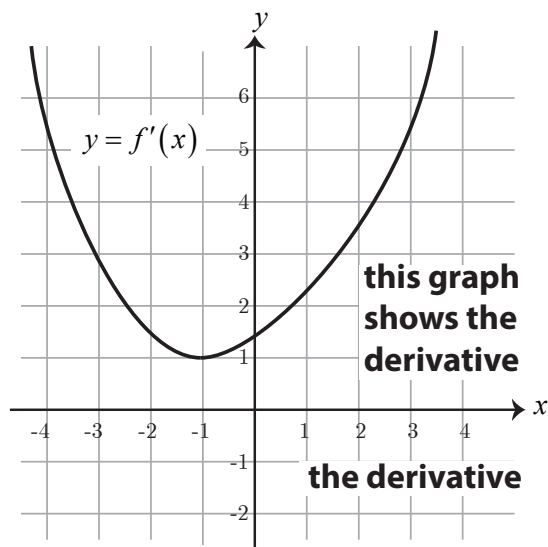
- (a)  $(-\infty, -1)$
- (b) nowhere
- (c)  $(-1, \infty)$
- (d)  $(-\infty, \infty)$
- (e)  $(1, \infty)$



- 
8. The following is the graph of the **derivative**,  $f'(x)$ , of the function  $f(x)$ .  
Where is the original function  $f(x)$  concave up?

**Possibilities:**

- (a)  $(-1, \infty)$
- (b)  $(1, \infty)$
- (c) nowhere
- (d)  $(-\infty, \infty)$
- (e)  $(-\infty, -1)$



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9. Find the critical numbers of the function  $f(x) = 2xe^{19x}$ .

**Possibilities:**

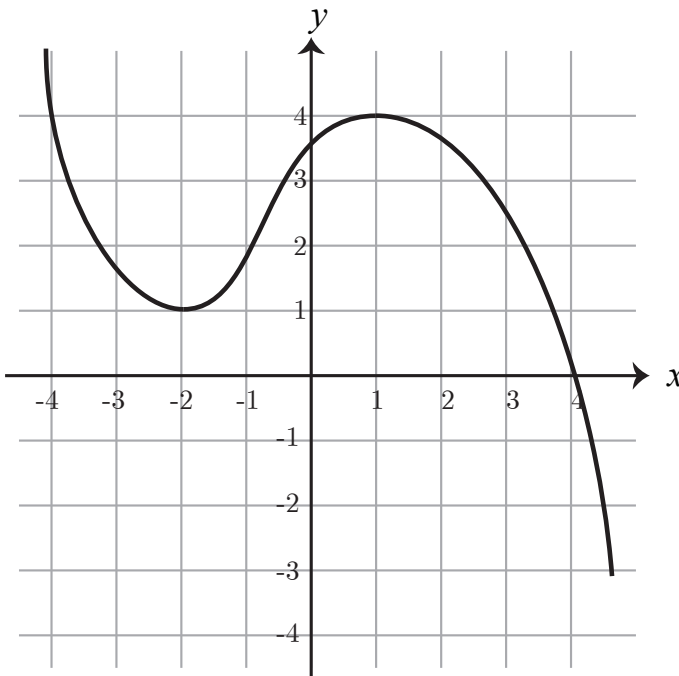
- (a)  $-\frac{2}{19}, 0$
- (b)  $0$
- (c)  $-\frac{1}{19}, 0, e^{19}$
- (d)  $-\frac{2}{19}$
- (e)  $-\frac{1}{19}$

---

10. Consider the graph of the original function,  $f(x)$ .  
For this function, what are the signs of  $f'(2)$  and  $f''(2)$ ?

**Possibilities:**

- (a)  $f'(2) < 0$  and  $f''(2) < 0$
- (b)  $f'(2) = 0$  and  $f''(2) < 0$
- (c)  $f'(2) > 0$  and  $f''(2) < 0$
- (d)  $f'(2) < 0$  and  $f''(2) > 0$
- (e)  $f'(2) > 0$  and  $f''(2) > 0$



- 
11. A farmer builds a rectangular pen with 3 vertical partitions (4 vertical sides) using 400 feet of fencing. What is the maximum possible total area of the pen?

**Possibilities:**

- (a) 10000
- (b) 4000
- (c) 5000
- (d) 20000
- (e) 400



- 
12. A car rental agency rents 180 cars per day at a rate of \$27 dollars per day. For each 1 dollar increase in the daily rate, 3 fewer cars are rented. At what rate should the cars be rented to produce maximum income (i.e., maximum daily revenue)?

**Possibilities:**

- (a) \$42.90 per day
- (b) \$43.10 per day
- (c) \$44.30 per day
- (d) \$43.70 per day
- (e) \$43.50 per day

- 
13. Given the function  $f(x) = \begin{cases} 0 & \text{if } x < -12 \\ 6 & \text{if } -12 \leq x < 0 \\ -60 & \text{if } 0 \leq x < 5 \\ 0 & \text{if } x \geq 5 \end{cases}$

evaluate the definite integral

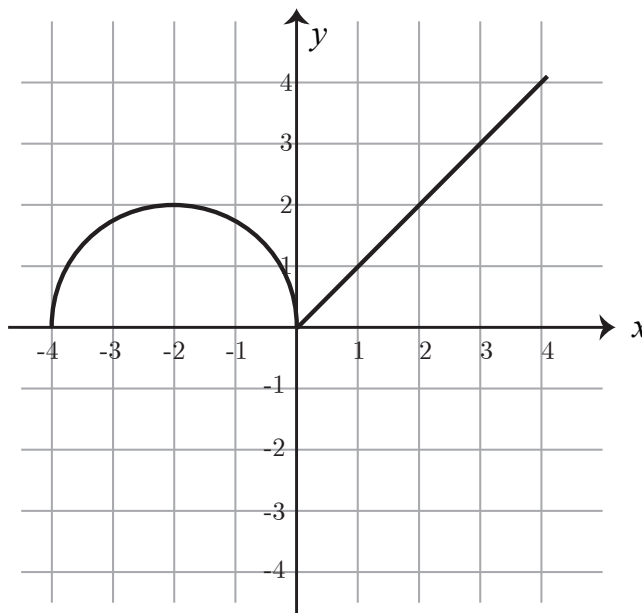
$$\int_{-12}^3 f(x) dx$$

**Possibilities:**

- (a)  $-108$   
(b)  $72$   
(c)  $-252$   
(d)  $252$   
(e)  $-228$
- 
14. The graph of  $y = f(x)$  shown below includes a semicircle and a straight line. Evaluate the definite integral  $\int_{-2}^4 f(x) dx$ .

**Possibilities:**

- (a)  $-\pi + 8$   
(b)  $\pi + 8$   
(c)  $-2\pi - 8$   
(d)  $2\pi + 8$   
(e)  $-2\pi + 8$





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15. Suppose that  $\int_1^{25} f(x) dx = 12$  and  $\int_7^{25} f(x) dx = 21$ . Find the value of  $\int_1^7 f(x) dx$ .

**Possibilities:**

(a)  $-33$

(b)  $33$

(c)  $9$

(d)  $-9$

(e)  $-\frac{3}{2}$

---

16. Suppose that  $\int_2^{24} f(x) dx = 8$ . Find the value of  $\int_2^{24} (3f(x) + 9) dx$ .

**Possibilities:**

(a)  $46$

(b)  $51$

(c)  $33$

(d)  $240$

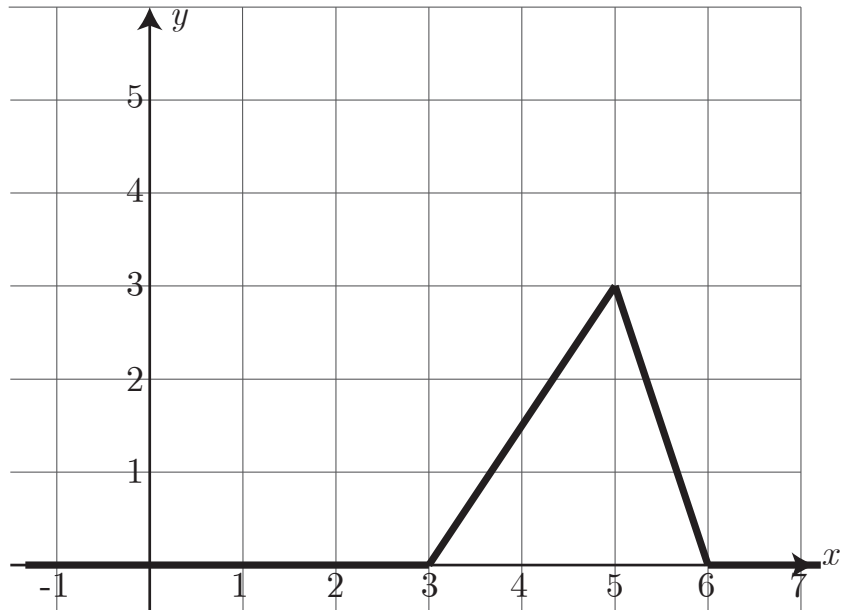
(e)  $222$

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- 
17. The graph of  $y = f(x)$  shown below consists of straight lines. Find the **average value** of  $f(x)$  on the interval  $[0, 6]$ .

**Possibilities:**

- (a)  $\frac{9}{4}$
- (b) 1
- (c)  $\frac{3}{4}$
- (d)  $\frac{9}{2}$
- (e)  $\frac{3}{2}$



- 
18. Estimate the area under the graph of  $y = x^2 + 6$  for  $x$  between 0 and 6, by using a partition that consists of 3 equal subintervals of  $[0, 6]$  and use the right endpoint of each subinterval as a sample point.

**Possibilities:**

- (a) 148
- (b) 74
- (c) 160
- (d) 76
- (e) 108

- 
19. Suppose you estimate the area under the graph of  $f(x) = \frac{1}{x}$  from  $x = 7$  to  $x = 25$  by adding the areas of the rectangles as follows: partition the interval into 6 equal subintervals and use the right endpoint of each interval to determine the height of the rectangle. What is the area of the 2<sup>nd</sup> rectangle?

**Possibilities:**

- (a)  $\frac{1}{13}$   
(b)  $-\ln(2) - \ln(5) + \ln(13)$   
(c)  $\frac{1230831}{1086800}$   
(d)  $\frac{3}{13}$   
(e)  $\frac{3}{10}$

- 
20. The rate (in liters per minute) at which water drains from a tank is recorded at half-minute intervals. Use the average of the left- and right-endpoint approximations to estimate the total amount of water drained during the first 2 minutes.

t min	0	.5	1	1.5	2
l/min	3	8	17	19	27

Use all five measurements in your estimate.

**Possibilities:**

- (a) 23.50 liters  
(b) 37.00 liters  
(c) 13.50 liters  
(d) 8.50 liters  
(e) 29.50 liters

## Some Formulas

### 1. Areas:

(a) Triangle  $A = \frac{bh}{2}$

(b) Circle  $A = \pi r^2$

(c) Rectangle  $A = lw$

(d) Trapezoid  $A = \frac{h_1 + h_2}{2} b$

### 2. Volumes:

(a) Rectangular Solid  $V = lwh$

(b) Sphere  $V = \frac{4}{3}\pi r^3$

(c) Cylinder  $V = \pi r^2 h$

(d) Cone  $V = \frac{1}{3}\pi r^2 h$

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