MA123 — Elem. Calculus Spring 2019
Exam 3 2019-4-11

Name: Solutions **Sec.:** _____

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) b c d e

You have two hours to do this exam. Please write your name and section number on this page.

GOOD LUCK!

3. a b c d **e**

4. a **b** c d e

5. a b c **d** e

6. a b c d **e**

7. a **b** c d e

8. **a** b c d e

9. a b c d **e**

10. **a** b c d e

11. a b **c** d e

12. a b c d **e**

13. **a** b c d e

14. a **b** c d e

15. a b c **d** e

16. a b c d **e**

17. a b **c** d e

18. **a** b c d e

19. a b c **d** e

20. a b c d **e**

For grading use:

Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(out of 100 points)

Spring 2019 Exam 3 Short Answer Questions

Write answers on this page. Your work must be clear and legible to be sure you will get full credit.

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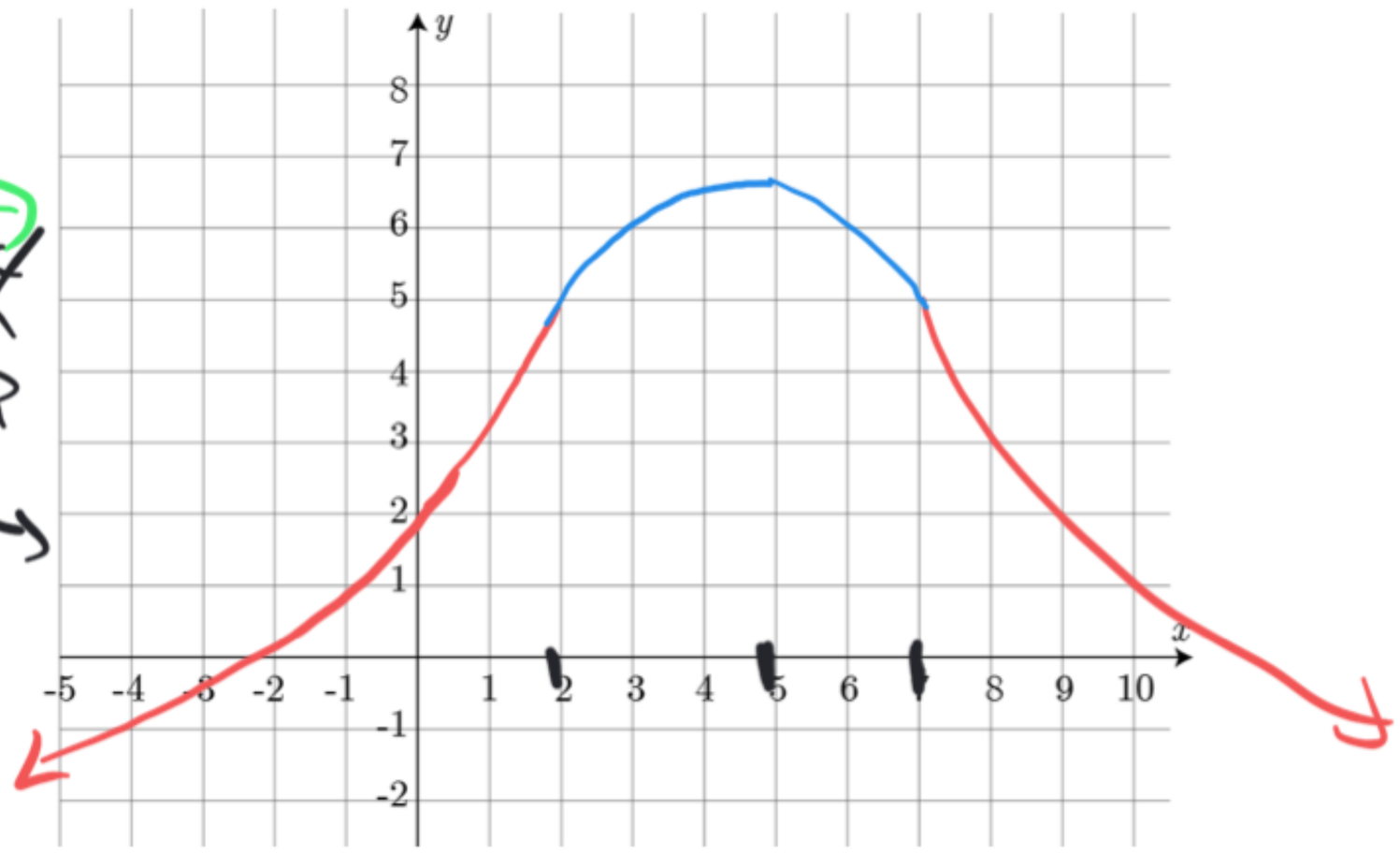
1. Sketch the graph of a **continuous** function $y = f(x)$ for which f is increasing on $(-\infty, 5)$, decreasing on $(5, \infty)$, $f''(x) > 0$ on $(-\infty, 2)$ and $(7, \infty)$; $f''(x) < 0$ on $(2, 7)$.

2 5 7

Inc/ Con. Up | Inc/ Con Down | Dec/ Con Down | Dec/ Con up



2. Suppose we know two nonnegative numbers x and y satisfying $4x + y = 13$. Find the maximum possible value of their product xy. *You must* ***CLEARLY USE CALCULUS*** *to find* ***and justify*** *your answer. Your final answer does* ***not*** *need to be simplified.*

Maximize the product $P = xy$

when $4x + y = 13$.

Solve for y

$y = -4x + 13$

Substitute in $P = xy$

$P = x(-4x + 13)$

$P = -4x^2 + 13x$

Find critical values by setting $P'(x) = 0$

$P'(x) = -8x + 13 \stackrel{?}{=} 0$

$x = \frac{13}{8}$

Evaluate product at critical value

$P = -4\left(\frac{13}{8}\right)^2 + 13\left(\frac{13}{8}\right)$

$= \frac{169}{16}$

Number line Test

$P'(0) > 0$ | $P'(2) < 0$

$\frac{13}{8}$ $\Rightarrow$ max

Maximum possible product: $\frac{169}{16}$

Name: _____

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

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3. Where is the function $f(t) = t^3 + 3t^2 - 72t + 4$ decreasing?

Possibilities:

(a) $t < -1$

(b) $t < -6$ and $t > 4$

(c) $f(t)$ is always decreasing

(d) $t > -1$

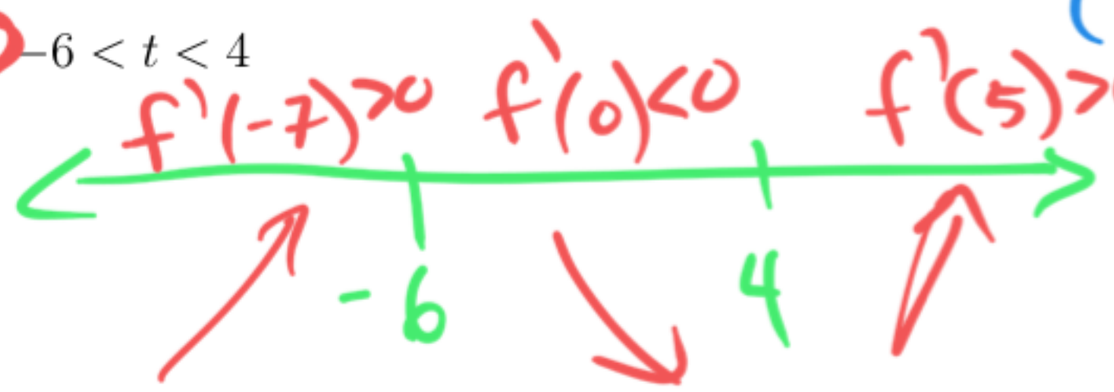
(e) $-6 < t < 4$

Set $f'(t) = 3t^2 + 6t - 72 \stackrel{?}{=} 0$

$3(t^2 + 2t - 24) = 0$

$(t-4)(t+6) = 0$

$\Rightarrow t = -6, 4$ are critical values.



decreasing on $(-6, 4)$

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4. Where is the function $f(t) = t^4 - 16t^3 - 9$ concave up?

Possibilities:

(a) $t > 12$

(b) $t < 0$ and $t > 8$

(c) $0 < t < 8$

(d) $t < 12$

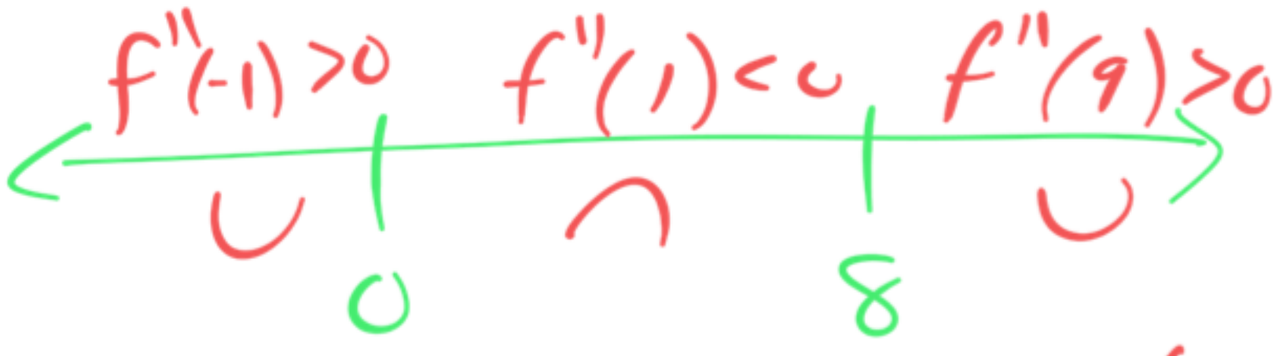
(e) $f(t)$ is always concave up

$f'(t) = 4t^3 - 48t^2$

Set $f''(t) = 12t^2 - 96t = 0$

$12t(t-8) = 0$

$\Rightarrow t = 0, 8$ are inflection values.



$\Rightarrow$ concave up on $(-\infty, 0) \cup (8, \infty)$

$t < 0$ or $t > 8$

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5. Suppose the derivative of $g(t)$ is $g'(t) = 11(t-2)^2(t-10)$. For t in which interval(s) is g increasing?

Possibilities:

(a) $(2, 10)$

(b) $(-\infty, 10)$

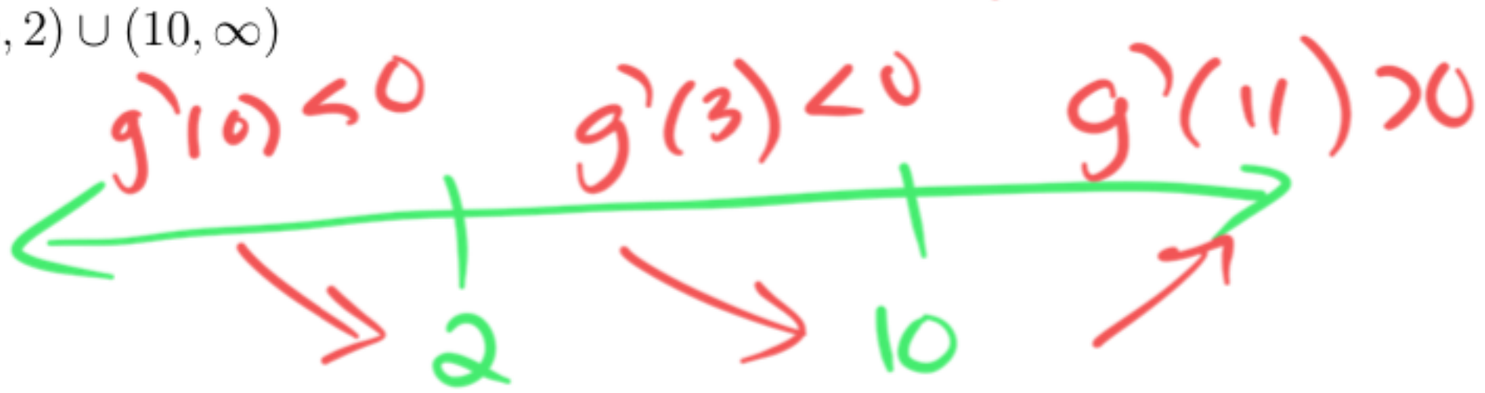
(c) $(2, 10) \cup (11, \infty)$

(d) $(10, \infty)$

(e) $(-\infty, 2) \cup (10, \infty)$

Set $g'(t) = 11(t-2)^2(t-10) = 0$

$\Rightarrow t = 2, 10$ are critical values



$\Rightarrow$ increasing on $(10, \infty)$

6. Suppose the derivative of $g(t)$ is $g'(t) = 11t^2 - 132t + 220$. For t in which interval(s) is g concave up?

Possibilities:

(a) $(2, 10)$

(b) $(-\infty, 2) \cup (10, \infty)$

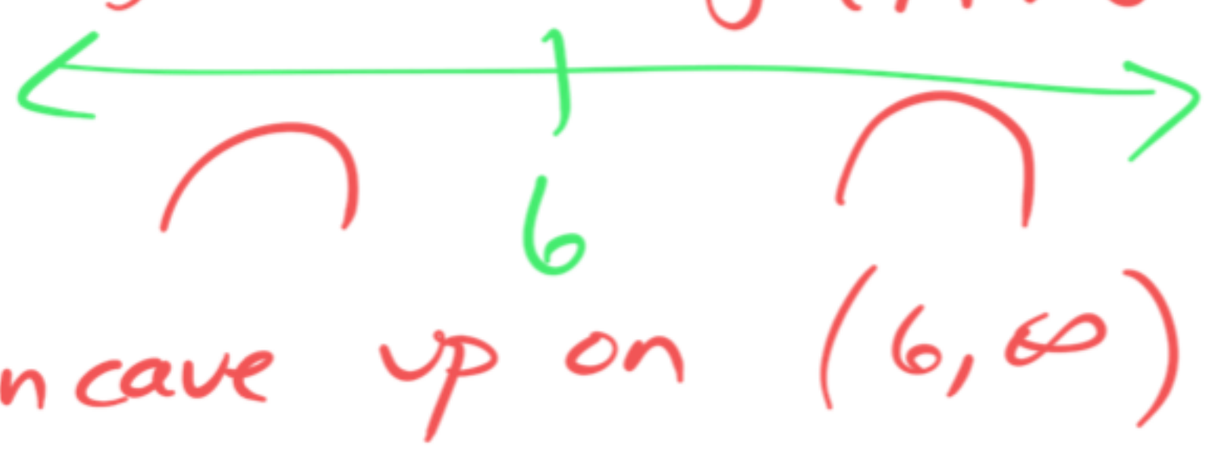
(c) $(2, 6) \cup (10, 11)$

(d) $(-\infty, 6)$

(e) $(6, \infty)$

Set $g''(t) = 22t - 132 = 0$

$\Rightarrow t = 6$ is the only inflection value



$\Rightarrow$ Concave up on $(6, \infty)$

7. The following is the graph of the **derivative**, $f'(x)$, of the function $f(x)$.
Where is the original function $f(x)$ decreasing?

$f'(x) < 0$

Possibilities:

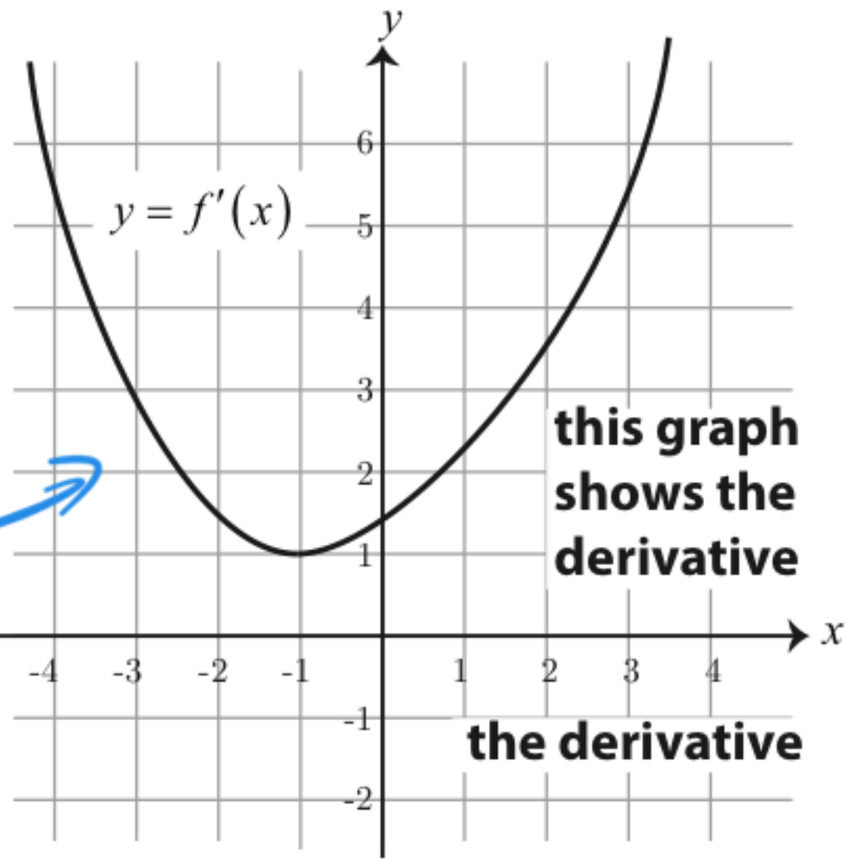
(a) $(-\infty, -1)$

(b) nowhere

(c) $(-1, \infty)$

(d) $(-\infty, \infty)$

(e) $(1, \infty)$



This graph is never below the x-axis $\Rightarrow$ $f'(x)$ is never negative

8. The following is the graph of the **derivative**, $f'(x)$, of the function $f(x)$.
Where is the original function $f(x)$ concave up?

$f'(x)$ is increasing

Possibilities:

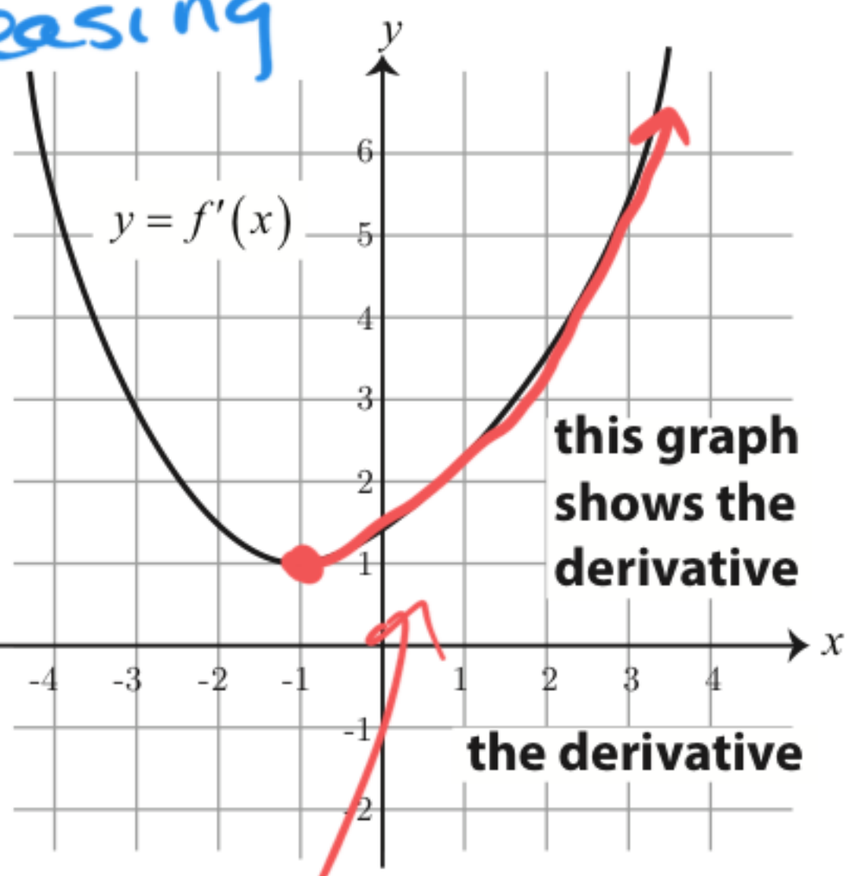
(a) $(-1, \infty)$

(b) $(1, \infty)$

(c) nowhere

(d) $(-\infty, \infty)$

(e) $(-\infty, -1)$



from -1 to ∞ $f'(x)$ is increasing

9. Find the critical numbers of the function $f(x) = 2xe^{19x}$.

Possibilities:

(a) $-\frac{2}{19}, 0$

(b) 0

(c) $-\frac{1}{19}, 0, e^{19}$

(d) $-\frac{2}{19}$

(e) $-\frac{1}{19}$

$f'(x) = (2x)'e^{19x} + 2x(e^{19x})'$

$= 2e^{19x} + 2xe^{19x}(19)$

Set equal to zero and solve for x

$2e^{19x} + 2e^{19x} \cdot 19x = 0$

$2e^{19x}(1 + 19x) = 0$

never zero

$\Rightarrow 1 + 19x = 0$

$x = -\frac{1}{19}$

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10. Consider the graph of the original function, $f(x)$.
For this function, what are the signs of $f'(2)$ and $f''(2)$?

Possibilities:

(a) $f'(2) < 0$ and $f''(2) < 0$

(b) $f'(2) = 0$ and $f''(2) < 0$

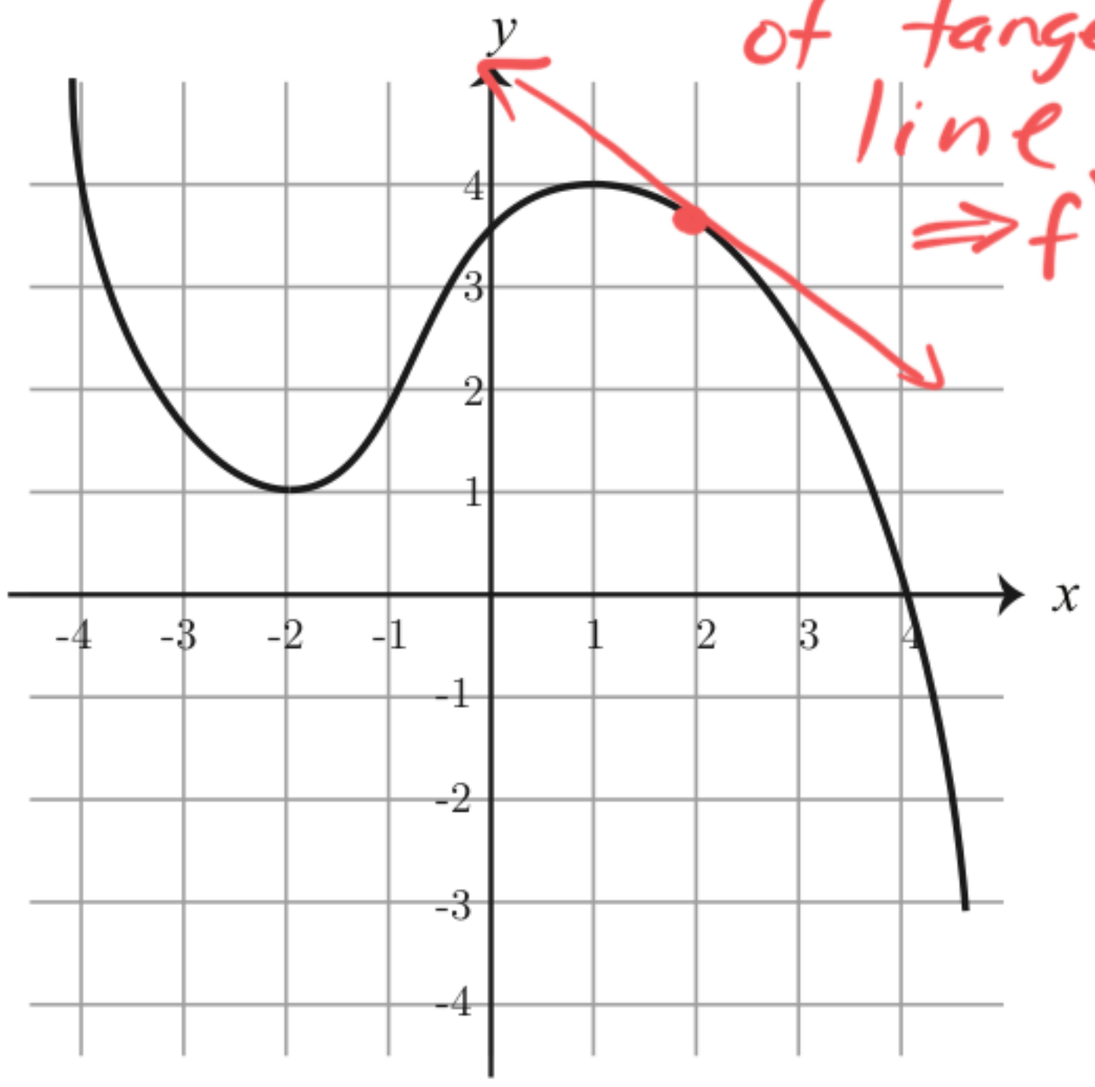
(c) $f'(2) > 0$ and $f''(2) < 0$

(d) $f'(2) < 0$ and $f''(2) > 0$

(e) $f'(2) > 0$ and $f''(2) > 0$

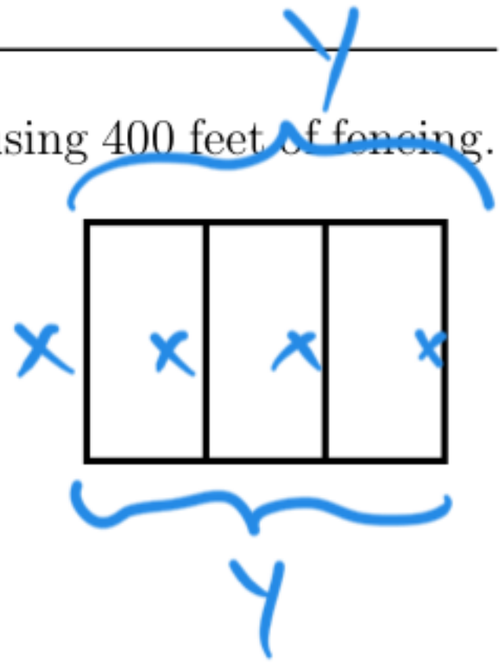
$f'(2)$ = slope of tangent line

$\Rightarrow f'(2) < 0$



$f''(2) < 0$ since $f(x)$ is concave down at $x = 2$.

11. A farmer builds a rectangular pen with 3 vertical partitions (4 vertical sides) using 400 feet of fencing. What is the maximum possible total area of the pen?



Possibilities:

(a) 10000

(b) 4000

(c) 5000

(d) 20000

(e) 400

$4x+2y=400$ Maximize $A=x\cdot y$

$\Rightarrow y=-2x+200$

$\Rightarrow A=x(-2x+200)=-2x^2+200x$

Solve $A'(x)=-4x+200=0 \Rightarrow x=50$

$\Rightarrow A=-2(50)^2+200(50)=5000$

$A'(49)>0$ / $A'(51)<0 \Rightarrow$ Max at $x=50$

12. A car rental agency rents 180 cars per day at a rate of \$27 dollars per day. For each 1 dollar increase in the daily rate, 3 fewer cars are rented. At what rate should the cars be rented to produce maximum income (i.e., maximum daily revenue)?

Possibilities:

(a) \$42.90 per day

(b) \$43.10 per day

(c) \$44.30 per day

(d) \$43.70 per day

(e) \$43.50 per day

($, cars)

(27, 180) (28, 177)

linear relationship

$\Rightarrow m=\frac{180-177}{27-28}=-3$

$\Rightarrow y-180=-3(x-27)$

$\Rightarrow y=-3x+261$

Revenue $=x\cdot y$

$R=x(-3x+261)$

$=-3x^2+261x$

$R'(x)=-6x+261$

$-6x+261=0$

$\Rightarrow x=43.5$

$R'(43)>0$ / $R'(44)>0$

43.5 max

13. Given the function $f(x) = \begin{cases} 0 & \text{if } x < -12 \\ 6 & \text{if } -12 \le x < 0 \\ -60 & \text{if } 0 \le x < 5 \\ 0 & \text{if } x \ge 5 \end{cases}$

evaluate the definite integral $\int_{-12}^{3} f(x)\,\mathrm{d}x$.

$$= \int_{-12}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx$$

$$= \int_{-12}^{0} 6\,dx + \int_{0}^{3} -60\,dx$$

$$= 6(12) - 60(3)$$

$$= 72 - 180$$

$$= -108$$

Possibilities:

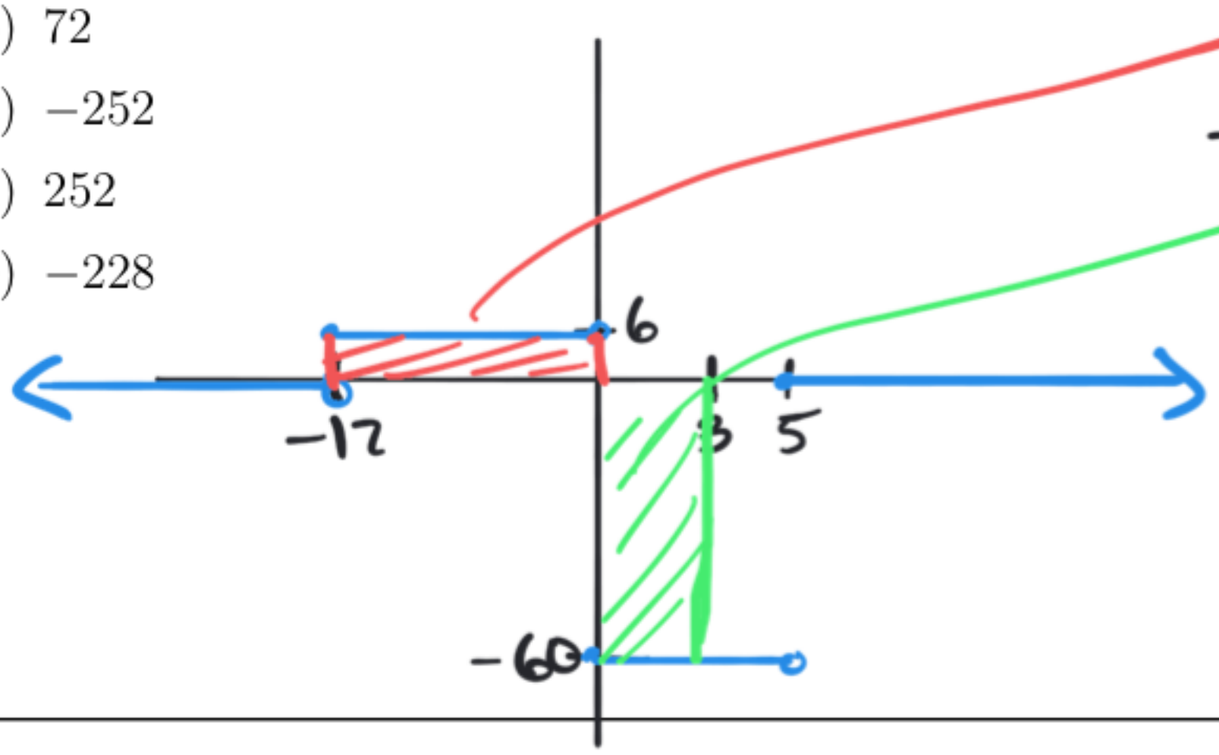
(a) -108

(b) 72

(c) -252

(d) 252

(e) -228



14. The graph of $y = f(x)$ shown below includes a semicircle and a straight line. Evaluate the definite integral $\int_{-2}^{4} f(x)\,\mathrm{d}x$.

$$= \int_{-2}^{0} f(x)\,dx + \int_{0}^{4} f(x)\,dx$$

$$= \text{Area of quarter circle} + \text{Area of triangle}$$

$$= \frac{\pi(2)^2}{4} + \frac{4(4)}{2}$$

$$= \pi + 8$$

Possibilities:

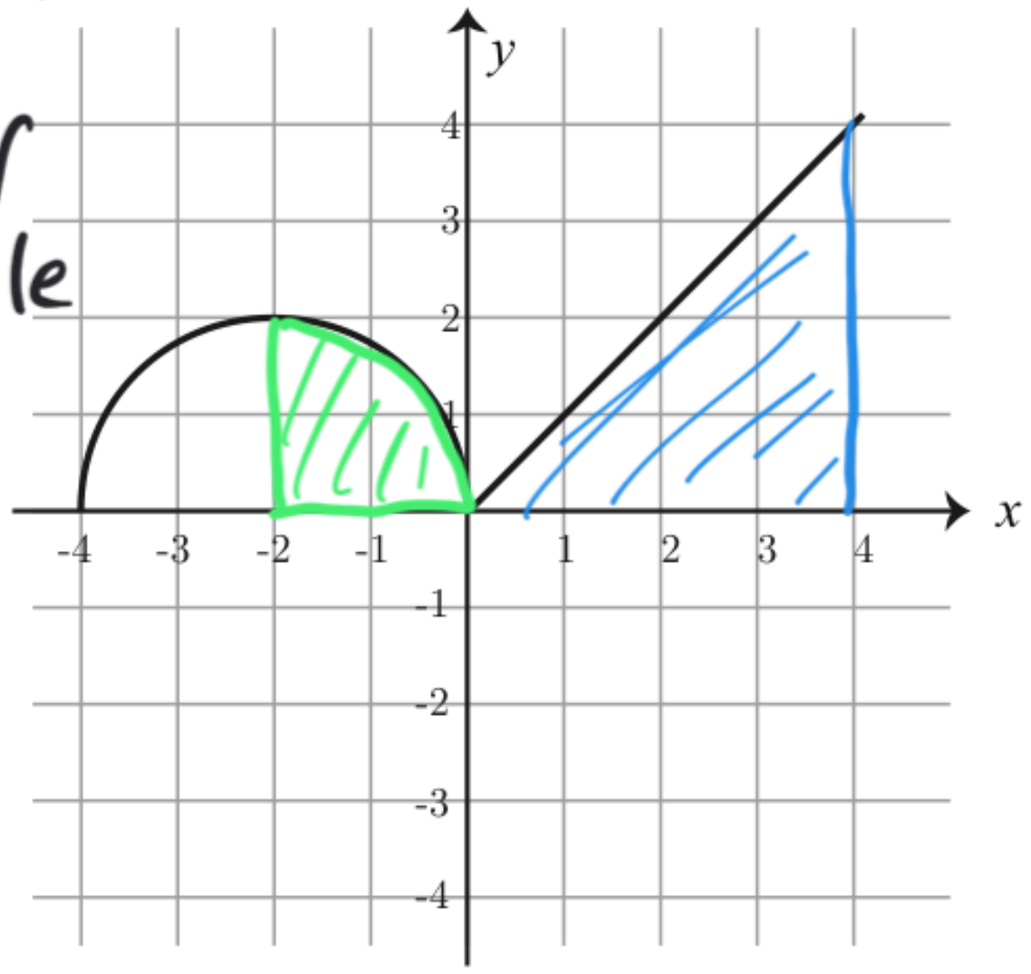
(a) $-\pi + 8$

(b) $\pi + 8$

(c) $-2\pi - 8$

(d) $2\pi + 8$

(e) $-2\pi + 8$



15. Suppose that $\int_1^{25} f(x)\,dx = 12$ and $\int_7^{25} f(x)\,dx = 21$. Find the value of $\int_1^7 f(x)\,dx$.

Possibilities:

(a) -33

(b) 33

(c) 9

(d) -9

(e) $-\frac{3}{2}$

$$\int_1^{25} f(x)\,dx = \int_1^7 f(x)\,dx + \int_7^{25} f(x)\,dx$$

$$12 = \int_1^7 f(x)\,dx + 21$$

$$\Longrightarrow \int_1^7 f(x)\,dx = 12 - 21 = -9$$

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16. Suppose that $\int_2^{24} f(x)\,dx = 8$. Find the value of $\int_2^{24} (3f(x) + 9)\,dx$.

Possibilities:

(a) 46

(b) 51

(c) 33

(d) 240

(e) 222

$$= 3\int_2^{24} f(x)\,dx + \int_2^{24} 9\,dx$$

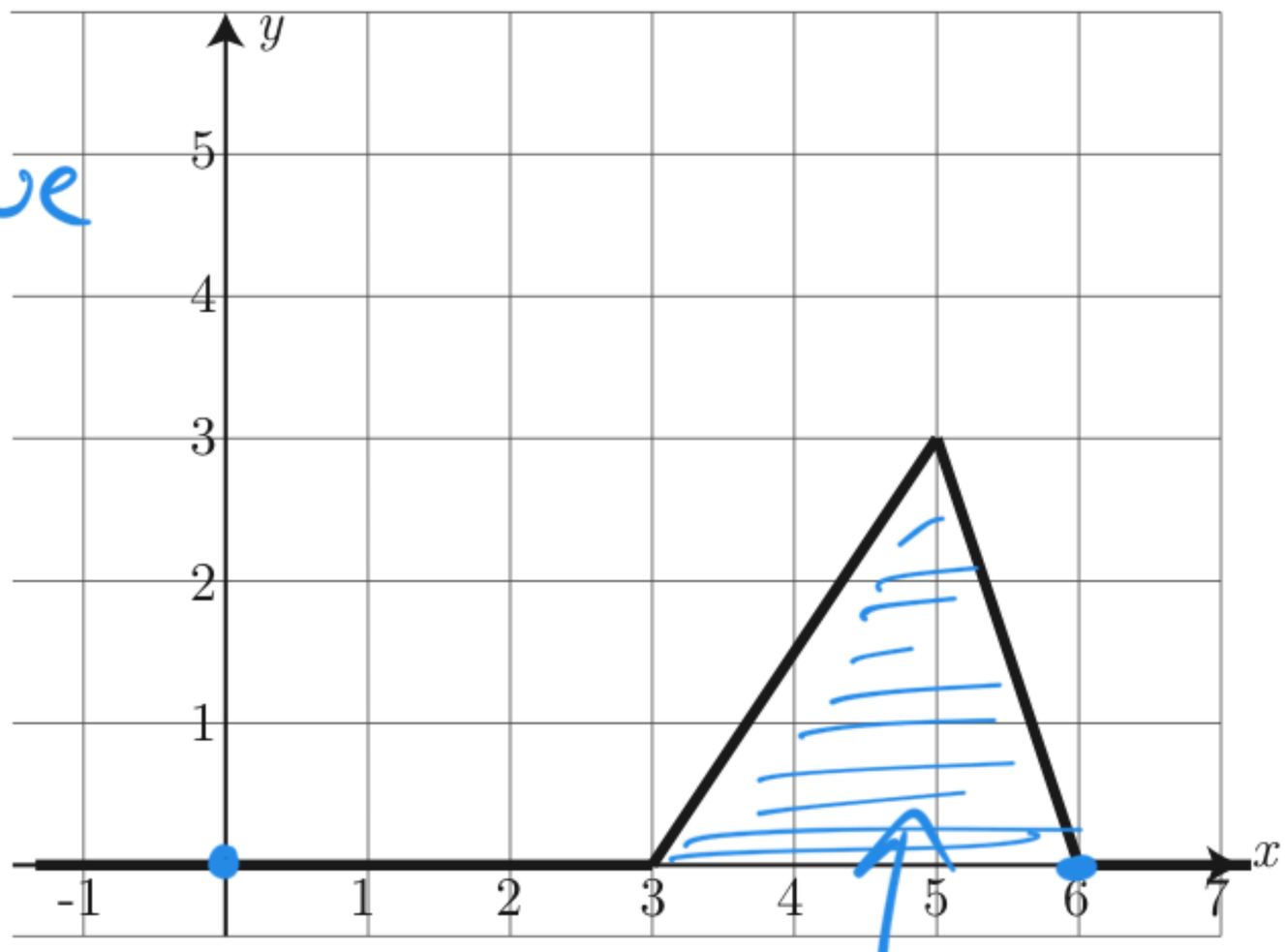
$$= 3(8) + 9(24-2)$$

$$= 24 + 9(22)$$

$$= 222$$

area of rectangle of height 9 and base 22.

17. The graph of $y = f(x)$ shown below consists of straight lines. Find the **average value** of $f(x)$ on the interval $[0, 6]$.



Possibilities:

(a) $\frac{9}{4}$

(b) 1

(c) $\frac{3}{4}$

(d) $\frac{9}{2}$

(e) $\frac{3}{2}$

Average value $= \frac{1}{6-0}\int_0^6 f(x)dx$

$= \frac{1}{6}\left(\frac{9}{2}\right)$

$= \frac{3}{4}$

Area $= \frac{3 \cdot 3}{2}$

$= 9/2$

18. Estimate the area under the graph of $y = x^2 + 6$ for x between 0 and 6, by using a partition that consists of 3 equal subintervals of $[0, 6]$ and use the right endpoint of each subinterval as a sample point.

$f(x) = x^2 + 6$

Possibilities:

(a) 148

(b) 74

(c) 160

(d) 76

(e) 108

Area $\approx$ base (sum of heights)

$= 2(f(2) + f(4) + f(6))$

$= 2(2^2+6 + 4^2+6 + 6^2+6)$

$= 148$

19. Suppose you estimate the area under the graph of $f(x) = \dfrac{1}{x}$ from $x = 7$ to $x = 25$ by adding the areas of the rectangles as follows: partition the interval into 6 equal subintervals and use the right endpoint of each interval to determine the height of the rectangle. What is the area of the 2^{nd} rectangle?

Possibilities:

(a) $\frac{1}{13}$

(b) $-\ln(2) - \ln(5) + \ln(13)$

(c) $\frac{1230831}{1086800}$

(d) $\frac{3}{13}$

(e) $\frac{3}{10}$

Area of 2nd rectangle
= base · height
= $3 \cdot f(13) = \frac{3}{13}$

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20. The rate (in liters per minute) at which water drains from a tank is recorded at half-minute intervals. Use the average of the left- and right-endpoint approximations to estimate the total amount of water drained during the first 2 minutes.

t min	0	.5	1	1.5	2
l/min	3	8	17	19	27

← height of each trapezoid is .5

Use all five measurements in your estimate.

Possibilities:

(a) 23.50 liters

(b) 37.00 liters

(c) 13.50 liters

(d) 8.50 liters

(e) 29.50 liters

Area = Sum of areas of trapezoids

$= \frac{1}{2}(3+8)(.5) + \frac{1}{2}(8+17)(.5) + \frac{1}{2}(17+19)(.5)$
$+ \frac{1}{2}(19+27)(.5)$

$= \frac{1}{4}(3+8+8+17+17+19+19+27) = \frac{1}{4}(118)$
$= 29.5$

Some Formulas

1. Areas:

(a) Triangle $A = \dfrac{bh}{2}$

(b) Circle $A = \pi r^2$

(c) Rectangle $A = lw$

(d) Trapezoid $A = \dfrac{h_1 + h_2}{2}\, b$

2. Volumes:

(a) Rectangular Solid $V = lwh$

(b) Sphere $V = \dfrac{4}{3}\pi r^3$

(c) Cylinder $V = \pi r^2 h$

(d) Cone $V = \dfrac{1}{3}\pi r^2 h$