MA123 — Elem. Calculus	Fall 2015	Name: SOLUTIONS	Sec.: ______
Final Exam	2015-12-17		

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name on this page, and at the top of page three.

GOOD LUCK!

3.	**(a)**	(b)	(c)	(d)	(e)	12.	(a)	(b)	(c)	**(d)**	(e)
4.	**(a)**	(b)	(c)	(d)	(e)	13.	(a)	(b)	**(c)**	(d)	(e)
5.	(a)	(b)	(c)	**(d)**	(e)	14.	(a)	(b)	(c)	(d)	**(e)**
6.	(a)	(b)	(c)	(d)	**(e)**	15.	**(a)**	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	**(e)**	16.	(a)	(b)	**(c)**	(d)	(e)
8.	**(a)**	(b)	(c)	(d)	(e)	17.	(a)	**(b)**	(c)	(d)	(e)
9.	(a)	(b)	(c)	**(d)**	(e)	18.	(a)	(b)	(c)	(d)	**(e)**
10.	(a)	**(b)**	(c)	(d)	(e)	19.	(a)	**(b)**	(c)	(d)	(e)
11.	(a)	(b)	**(c)**	(d)	(e)	20.	**(a)**	(b)	(c)	(d)	(e)

For grading use:

Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(out of 100 points)

Fall 2015 Exam 4 Short Answer Questions

Write answers on this page. You must show ***appropriate legible work*** *to be sure you will get full credit.*

1. Evaluate $\int_0^T \sqrt{3x+10}\,dx$. Show steps clearly and circle your answer. You do **NOT** need to simplify your final answer.

use u-substitution

let $u = 3x+10$

then $\frac{du}{dx} = 3$ so $dx = \frac{1}{3}du$

if $x=0$ then $u = 3(0)+10 = 10$

and if $x=T$ then $u = 3T+10$

so $\int_0^T \sqrt{3x+10}\,dx = \int_{10}^{3T+10} \sqrt{u}\cdot\frac{1}{3}du = \frac{1}{3}\int_{10}^{3T+10} u^{1/2}\,du$

$= \frac{1}{3}\left(\frac{2}{3}u^{3/2}\right)\Big|_{10}^{3T+10}$

$= \boxed{\frac{2}{9}(3T+10)^{3/2} - \frac{2}{9}(10)^{3/2}}$

2. If the derivative of a function $g(x)$ is given by $g'(x) = (x+2)(x+5)$, find interval(s) where $g(x)$ is concave up. Show work clearly.

$g(x)$ is concave up where $g''(x) > 0$

$g'(x) = (x+2)(x+5) = x^2+7x+10$

$g''(x) = 2x+7$

$2x+7=0$ when $2x=-7$

$x = -7/2$

$g''(x)$ concave down $-$ | $+$ concave up

$-7/2$

check -4 check 0

Answer: $\boxed{(-7/2, \infty)}$

Name: SOLUTIONS

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

3. Find the limit as n tends to infinity. Here C is a fixed real number.

$$\lim_{n\to\infty} \frac{(Cn+1)^2}{4n^2+3n+7}$$

terms with highest order are most important!

$$= \lim_{n\to\infty} \frac{C^2 n^2}{4n^2} = \lim_{n\to\infty} \frac{C^2}{4} = \frac{C^2}{4}$$

Possibilities:

(a) $\frac{1}{4}C^2$

(b) ∞

(c) $\frac{1}{16}C^2$

(d) $\frac{1}{14}C$

(e) 0

4. Evaluate the limit as n tends to infinity. Note: you will have to use some of the summation formulas (see formula sheet on backpage) to simplify.

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{6k^2}{n^2}$$

$$= \lim_{n\to\infty} \frac{1}{n} \cdot \frac{6}{n^2} \sum_{k=1}^{n} k^2$$

$$= \lim_{n\to\infty} \frac{6}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$ highest order terms most important!

$$= \lim_{n\to\infty} \frac{12n^3}{6n^3} = \lim_{n\to\infty} 2 = 2$$

Possibilities:

(a) 2

(b) 1

(c) 3

(d) 4

(e) 5

5. Assuming $x > 0$, evaluate the definite integral

$$\int_7^x \frac{6}{t}\ dt$$

$$= 6\int_7^x \frac{1}{t}\ dt = 6 \ln|t| \Big|_7^x$$

$$= 6 \ln|x| - 6 \ln|7|$$

Possibilities:

(a) $-\frac{6}{x^2} + \frac{6}{49}$

(b) $12\sqrt{x} - 12\sqrt{7}$

(c) $6\sqrt{x}$

(d) $6\ln(|x|) - 6\ln(7)$

(e) $\frac{6}{\frac{1}{2}x^2} - \frac{12}{49}$

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6. Use the Fundamental Theorem of Calculus to compute the derivative, $F'(x)$, of $F(x)$, if

$$F(x) = \int_1^{x+9} (t^2 + 7t + 3)\ dt$$

FTC says

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

So

Possibilities:

(a) $2x + 7$

(b) $\frac{1}{3}x^3 + \frac{7}{2}x^2 + 3x - \left(\frac{1}{3}1^3 + \frac{7}{2}1^2 + 3(1)\right)$

(c) $x^2 + 7x + 3$

(d) $\frac{1}{3}(x+9)^3 + \frac{7}{2}(x+9)^2 + 3(x+9) - \left(\frac{1}{3}1^3 + \frac{7}{2}1^2 + 3(1)\right)$

(e) $(x+9)^2 + 7(x+9) + 3$

$$\frac{d}{dx}\int_1^{x+9} (t^2 + 7t + 3)\,dt = (x+9)^2 + 7(x+9) + 3$$

7. Find the value of x at which

$$F(x) = \int_3^x \left(-t^4 - t^2 - 7\right)\ dt$$

takes its minimum value on the interval $[9, 800]$.

Possibilities:

(a) 3

(b) 97

(c) $\frac{393}{5}$

(d) 9

(e) 800

$F'(x) = -x^4 - x^2 - 7$ by FTC

$-x^4 - x^2 - 7 < 0$ for all x

thus $F(x)$ is always decreasing.

So minimum occurs at the rightmost endpoint $x = 800$

8. Evaluate the integral

$$\int_0^x (7t+9)^{15}\ dt$$

Possibilities:

(a) $\frac{1}{7(16)}(7x+9)^{16} - \frac{9^{16}}{7(16)}$

(b) $\frac{1}{15}(7x+9)^{15} - \frac{9^{15}}{15}$

(c) $\frac{1}{16}(7x+9)^{16} - \frac{9^{16}}{16}$

(d) $16(7x+9)^{16} - 15 \cdot 9^{16}$

(e) $\frac{1}{16}x^{16} - \frac{9^{16}}{16}$

let $u = 7t+9$

then $\frac{du}{dt} = 7$ so $dt = \frac{1}{7} du$

if $t = 0$ then $u = 7(0)+9 = 9$

$t = x$ then $u = 7x+9$

$$\int_0^x (7t+9)^{15}\, dt = \int_9^{7x+9} u^{15} \cdot \frac{1}{7}\, du$$

$$= \frac{1}{7}\int_9^{7x+9} u^{15}\, du$$

$$= \frac{1}{7}\left(\frac{1}{16}u^{16}\right)\Big|_9^{7x+9}$$

$$= \frac{1}{7}\cdot\frac{1}{16}\left[(7x+9)^{16} - (9)^{16}\right]$$

9. Suppose a rock is dropped from a lunar cliff. After t seconds, its speed in feet per second is $v(t) = \frac{53}{10}t$, at least until it lands. If the rock lands after 9 seconds, how high (in feet) is the cliff?

Possibilities:

(a) $\frac{9}{2}$ feet

(b) $\frac{53}{90}$ feet

(c) 9 feet

(d) $\frac{4293}{20}$ feet

(e) $\frac{477}{10}$ feet

$$\int_0^9 \left(\frac{53}{10}t\right) dt$$

$$= \left(\frac{1}{2} \cdot \frac{53}{10} t^2\right)\Big|_0^9$$

$$= \frac{53}{20} \cdot 9^2 - \frac{53}{20} \cdot 0^2$$

$$= \frac{4293}{20}$$

10. Compute $\lim_{t \to 2} \frac{t^2 + 6t - 16}{t^2 + 2t - 8}$

Possibilities:

(a) $\frac{4}{3}$

(b) $\frac{5}{3}$

(c) 2

(d) $\frac{7}{3}$

(e) The limit does not exist.

substitution gives us "$\frac{0}{0}$" - must do more work!

$$\lim_{t \to 2} \frac{(t+8)\cancel{(t-2)}}{(t+4)\cancel{(t-2)}}$$

$$= \lim_{t \to 2} \frac{t+8}{t+4} = \frac{2+8}{2+4} = \frac{10}{6} = \frac{5}{3}$$

11. Find the average rate of change of $f(x) = \sqrt{x}$ from $x = 49$ to $x = 81$.

Possibilities:

(a) $\dfrac{\log(49) + \log(81)}{2}$

(b) $\dfrac{1}{2}(81)^{-1/2} - \dfrac{1}{2}(49)^{-1/2}$

(c) $\dfrac{\sqrt{81} - \sqrt{49}}{81 - 49}$

(d) $\dfrac{\sqrt{81} - \sqrt{49}}{\sqrt{49} - \sqrt{81}}$

(e) $\dfrac{1}{49} - \dfrac{1}{81}$

$$AROC = \frac{f(81) - f(49)}{81 - 49}$$

$$= \frac{\sqrt{81} - \sqrt{49}}{32} = \frac{9 - 7}{32} = \frac{2}{32} = \frac{1}{16}$$

12. If an amount of x dollars is invested at 5% interest compounded continuously, and at the end of 2 years the value of the investment is \$6000, find x.

Possibilities:

(a) \$2853.69

(b) \$600

(c) \$5000

(d) \$5429.02

(e) \$6631.02

$$P = P_0 e^{rt}$$

$$6000 = x e^{.05(2)}$$

$$6000 = x e^{.1}$$

$$\frac{6000}{e^{.1}} = x$$

$$\$5429.02 \approx x$$

13. The tangent line to the graph of f at $x = 7$ has equation $y = 9(x - 7) + 3$. Find $f(7)$ and $f'(7)$.

Possibilities:

(a) $f(3) = 7, \quad f'(3) = 9$

(b) $f(7) = 9, \quad f'(7) = 3$

(c) $f(7) = 3, \quad f'(7) = 9$

(d) $f(3) = 9, \quad f'(3) = 7$

(e) $f(9) = 3, \quad f'(9) = 7$

$y = 9(x-7) + 3$

$y - 3 = 9(x-7)$

point slope form gives us

$m = f'(7) = 9$

point $(7,3)$ so $f(7) = 3$

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14. The graph of $y = f(x)$ is shown below. The function is differentiable, except at $x =$

Possibilities:

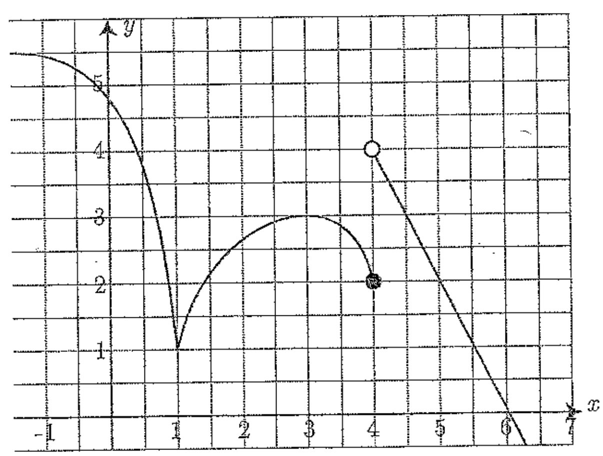
(a) x=1, x=3, and x=4

(b) x=1 only

(c) x=1, x=3, x=4, and x=6

(d) x=4 only

(e) x=1 and x=4



look for places where

— holes, skips, jumps exist $(x=4)$

— "sharp turns" occur $(x=1)$

— Vertical asymptotes are present (none exist here)

15. If $f(x) = 8x^5 + 4x$ then find the second derivative $f''(x)$:

Power rule twice!

$f'(x) = 40x^4 + 4$

$f''(x) = 160x^3$

Possibilities:

(a) $160x^3$

(b) $40x^4 + 4$

(c) $40x^4 + 80x^3 + 80x^2 + 40x + 12$

(d) $200x^5$

(e) $160x^3 + 80x$

16. Suppose $F(x) = g(x) \cdot h(x+2)$. If $g(0) = 8$, $g'(0) = 4$, $h(0) = 9$, $h'(0) = 7$, $h(2) = 5$, and $h'(2) = 3$, find $F'(0)$.

power rule

chain rule

$F'(x) = g'(x) \cdot h(x+2) + g(x) h'(x+2)(1)$

$F'(0) = g'(0) \cdot h(2) + g(0) h'(2)$

$= 4 \cdot 5 + 8 \cdot 3$

$= 20 + 24 = 44$

Possibilities:

(a) 95

(b) 68

(c) 44

(d) 158

(e) 36

17. Find the derivative, $f'(x)$, if $f(x) = e^{x^3+3x^2+4x}$.

Possibilities:

(a) e^{3x^2+6x+4}

(b) $(3x^2+6x+4)e^{x^3+3x^2+4x}$

(c) $\ln(x^3+3x^2+4x)$

(d) $(3x^2+6x+4)e^x$

(e) $\dfrac{3x^2+6x+4}{x^3+3x^2+4x}$

chain rule

$f'(x) = e^{x^3+3x^2+4x}(3x^2+6x+4)$

18. Suppose the derivative of $g(t)$ is $g'(t) = 9(t-7)(t-3)(t-8)$. For t in which interval(s) is g increasing?

Possibilities:

(a) $(-\infty, 6-\frac{1}{3}\sqrt{21}) \cup (6+\frac{1}{3}\sqrt{21}, \infty)$

(b) $(9,3) \cup (7,8)$

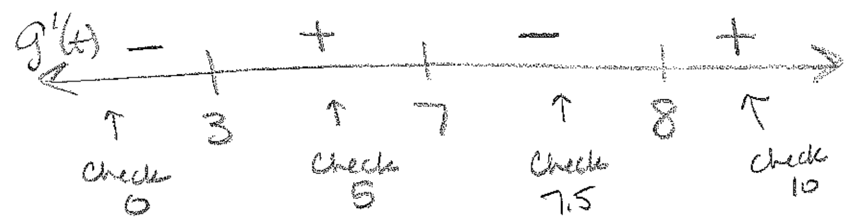
(c) $(6-\frac{1}{3}\sqrt{21}, 6+\frac{1}{3}\sqrt{21})$

(d) $(-\infty,3) \cup (7,8)$

(e) $(3,7) \cup (8,\infty)$

g(t) is increasing when $g'(t) > 0$

$g'(t) = 9(t-7)(t-3)(t-8) = 0$ when $t = 7, 3, 8$



g(t) increasing $(3,7) \cup (8,\infty)$

19. Find the area of the largest rectangle whose sides are parallel to the coordinate axes, whose bottom-left corner is at $(0,0)$ and whose top-right corner is on the graph of $y = 6x - x^2$.

Possibilities:

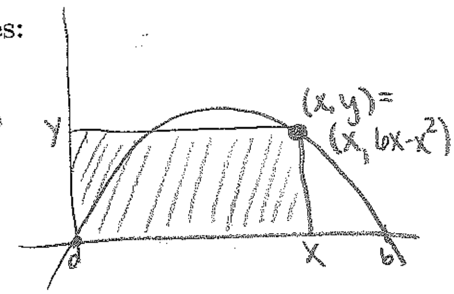
(a) 0

(b) 32

(c) 30

(d) 27

(e) 3

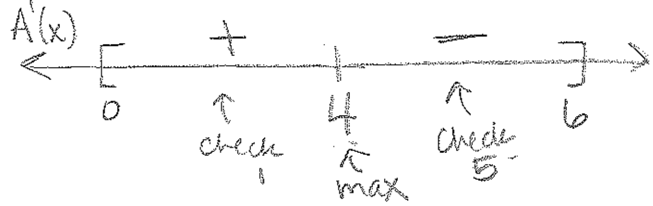


$A = x \cdot y = x(6x - x^2) = 6x^2 - x^3$ where $x \in [0,6]$

$A'(x) = 12x - 3x^2$

$12x - 3x^2 = 3x(4-x) = 0$

when $x = 0, 4$



max occurs at $x=4$, $A(4) = 4(6(4) - 4^2) = 4(8) = 32$

20. A farmer currently has harvested 290 bushels of kale that are currently worth \$10.51 per bushel. The way things are going, he expects to be harvesting 3.00 bushels per day, and expects the price to be increasing at \$0.25 per bushel per day. What is the instantaneous rate of change (measured in dollars per day) of the total value of his kale?

Possibilities:

(a) \$104.03 per day

(b) \$104.04 per day

(c) \$104.05 per day

(d) \$104.06 per day

(e) \$104.07 per day

V = total value

n = # of bushels

P = price / bushel

$V = n \cdot p$

differentiate! Product rule

$$\frac{dV}{dt} = n \cdot \frac{dp}{dt} + \frac{dn}{dt} \cdot p$$

$= (290)(.25) + (3)(10.51)$

$= 72.5 + 31.53$

$= \$104.03$

Some Formulas

1. Summation formulas:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

2. Areas:

   (a) Triangle $A = \frac{bh}{2}$

   (b) Circle $A = \pi r^2$

   (c) Rectangle $A = lw$

   (d) Trapezoid $A = \frac{h_1 + h_2}{2} b$

3. Volumes:

   (a) Rectangular Solid $V = lwh$

   (b) Sphere $V = \frac{4}{3}\pi r^3$

   (c) Cylinder $V = \pi r^2 h$

   (d) Cone $V = \frac{1}{3}\pi r^2 h$

4. Distance:

   (a) Distance between (x_1, y_1) and (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$