MA123 — **Elem. Calculus**	Fall 2016	Name: SOLUTIONS Sec.: ______
Final Exam	2016-12-14	

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and twenty multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name on this page, and at the top of page three.

GOOD LUCK!

3.	**(a)**	(b)	(c)	(d)	(e)	**13.**	(a)	(b)	**(c)**	(d)	(e)
4.	(a)	(b)	(c)	(d)	**(e)**	**14.**	(a)	(b)	(c)	(d)	**(e)**
5.	(a)	**(b)**	(c)	(d)	(e)	**15.**	**(a)**	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	**(e)**	**16.**	(a)	(b)	**(c)**	(d)	(e)
7.	**(a)**	(b)	(c)	(d)	(e)	**17.**	(a)	**(b)**	(c)	(d)	(e)
8.	**(a)**	(b)	(c)	(d)	(e)	**18.**	(a)	(b)	(c)	(d)	**(e)**
9.	(a)	(b)	(c)	**(d)**	(e)	**19.**	(a)	**(b)**	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	**(e)**	**20.**	**(a)**	(b)	(c)	(d)	(e)
11.	(a)	(b)	**(c)**	(d)	(e)	**21.**	(a)	(b)	**(c)**	(d)	(e)
12.	(a)	(b)	(c)	**(d)**	(e)	**22.**	(a)	(b)	(c)	**(d)**	(e)

For grading use:

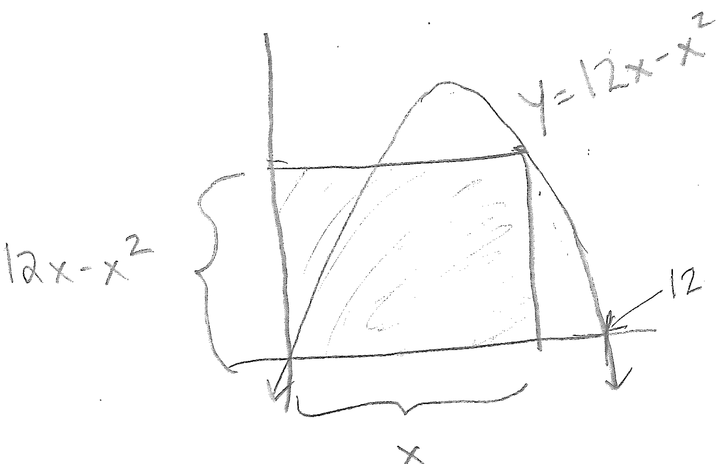
Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(max 110 points)

Fall 2016 Exam 4 Short Answer Questions

Write answers on this page. You must show appropriate legible work to be sure you will get full credit.

6 pts 1. Find the maximum area of a rectangle whose sides are parallel to the coordinate axes, whose bottom-left corner is at (0, 0), and whose top-right corner is on the graph of $y = 12x - x^2$. *You must **clearly use calculus** to find **and justify** your answer.*



Area of the rectangle

$= A(x) = b \cdot h = x(12x - x^2)$

x is a real number in $[0,12]$

Maximize $A(x) = 12x^2 - x^3 \Rightarrow$ Check when $A'(x) = 0$

$A'(x) = 24x - 3x^2 = 3x(8-x) = 0$ when $x = 0$ or $x = 8$

Test $A(0) = 0$
$A(12) = 0$
$A(8) = 256$

Maximum area: 256

4 pts 2. Evaluate $\int_0^T \left(e^x + x^{11} + 2\right) dx$. Show steps clearly and circle your final answer. You do **NOT** need to simplify your final answer.

$$\int_0^T (e^x + x^{11} + 2)\,dx = \int_0^T e^x\,dx + \int_0^T x^{11}\,dx + \int_0^T 2\,dx$$

$$= e^x\Big|_0^T + \frac{x^{12}}{12}\Big|_0^T + 2x\Big|_0^T$$

$$= e^T - e^0 + \frac{T^{12}}{12} - 0 + 2T - 0$$

$$= \boxed{e^T - 1 + \frac{T^{12}}{12} + 2T}$$

Name: ____________________

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

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3. A train travels in a straight westward direction along a track. The speed of the train varies, but it is measured at regular time intervals of 1/10 hour. The measurements for the first half hour are:

time	0	.1	.2	.3	.4	.5
speed	0	7	10	15	19	25

Estimate the total distance (in miles) traveled by the train during the first half hour by assuming the speed is a linear function of t on the subintervals. The speed in the table is given in miles per hour. Use all six speed measurements in your estimate.

Possibilities:

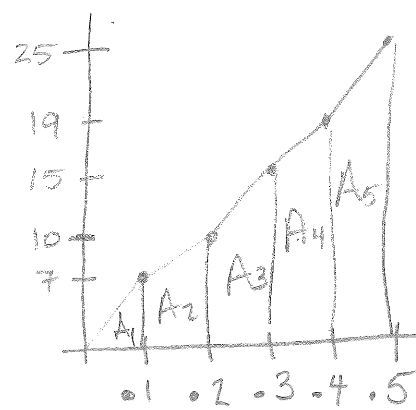
(a) 6.35 miles

(b) 3.50 miles

(c) 7.50 miles

(d) 7.60 miles

(e) 12.50 miles



Distace = Area under graph.

$A_1 = \frac{(.1)(7)}{2} = .35$ $A_4 = \frac{(.1)(15+19)}{2} = 1.7$

$A_2 = \frac{(.1)(7+10)}{2} = .85$ $A_5 = \frac{(.1)(19+25)}{2} = 2.2$

$A_3 = \frac{(.1)(10+15)}{2} = 1.25$

$\boxed{\text{SUM} = 6.35}$

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4. Suppose that the average value of $f(x)$ on $[6, 20]$ is 62. Find the value of $\int_6^{20} f(x)\,dx$.

Possibilities:

(a) 11284

(b) 434

(c) 898

(d) 1736

(e) 868

Average value of f on $[a,b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$

$\Rightarrow \frac{1}{20-6}\int_6^{20} f(x)\,dx = 62$

$\Rightarrow \frac{1}{14}\int_6^{20} f(x)\,dx = 62 \Rightarrow \int_6^{20} f(x)\,dx = 62 \cdot 14 = 868$

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5. Evaluate the definite integral

$$\int_7^x \frac{4}{\sqrt{t}}\, dt$$

Possibilities:

(a) $2\sqrt{x} - 2\sqrt{7}$

(b) $8\sqrt{x} - 8\sqrt{7}$

(c) $4\sqrt{x} - 4\sqrt{7}$

(d) $4\sqrt{x}$

(e) $\frac{4}{\sqrt{x}} - \frac{4}{\sqrt{7}}$

$$\int_7^x \frac{4}{\sqrt{t}}\, dt = \int_7^x 4t^{-1/2}\, dt$$

$$= \frac{4\, t^{1/2}}{1/2} \Big|_7^x$$

$$= 8t^{1/2} \Big|_7^x$$

$$= 8x^{1/2} - 8 \cdot 7^{1/2}$$

$$= 8\sqrt{x} - 8\sqrt{7}$$

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6. Find the average value of $f(x) = x^3$ over [1,23].

Possibilities:

(a) 69960.00

(b) 4055.33

(c) 184.33

(d) 265.00

(e) 3180.00

Average value of $f(x)$ over $[a,b]$

$$= \frac{1}{b-a} \int_a^b f(x)\,dx$$

$$\Rightarrow \frac{1}{23-1} \int_1^{23} x^3\,dx = \frac{1}{22}\left(\frac{x^4}{4}\Big|_1^{23}\right)$$

$$= \frac{1}{22}\left(\frac{23^4}{4} - \frac{1^4}{4}\right) = 3180$$

7. Let

$$F(x) = \int_0^x (t^2 + t - 42)\ dt$$

For which positive value of x does $F'(x) = 0$?

Possibilities:

(a) 6

(b) 42

(c) 7

(d) 48

(e) $-\frac{1}{2}$

By the Fundament Theorem of Calculus

$$\frac{d}{dx}(F(x)) = \frac{d}{dx}\left(\int_0^x (t^2+t-42)\,dt\right)$$

$$= x^2 + x - 42$$

$\Rightarrow F'(x) = 0$ when $x^2 + x - 42 = 0 \Rightarrow (x+7)(x-6) = 0$

$\Rightarrow x = -7$ or $x = 6$

$x = 6$ is positive value

8. Use the Fundamental Theorem of Calculus to compute the derivative, $F'(x)$, of $F(x)$, if

$$F(x) = \int_1^{x+9} (t^2 + 6t + 4)\ dt$$

$$\rightarrow F'(x) = \frac{d}{dx}\left(\int_1^{x+9} (t^2+6t+4)\,dt\right)$$

Possibilities:

(a) $(x+9)^2 + 6(x+9) + 4$

(b) $\frac{1}{3}(x+9)^3 + \frac{6}{2}(x+9)^2 + 4(x+9) - \left(\frac{1}{3}1^3 + \frac{6}{2}1^2 + 4(1)\right)$ $= \left((x+9)^2 + 6(x+9) + 4\right)\cdot(x+9)'$

(c) $\frac{1}{3}x^3 + \frac{6}{2}x^2 + 4x - \left(\frac{1}{3}1^3 + \frac{6}{2}1^2 + 4(1)\right)$

(d) $x^2 + 6x + 4$ $= \left((x+9)^2 + 6(x+9) + 4\right)\cdot 1$

(e) $2x + 6$

need to multiply by the derivative of $(x+9)$ by the FTOC

but in this case it was 1.

9. Evaluate the integral

$$\int_0^x (6t+9)^{20}\, dt$$

Need to use u-substitution

Let $u = 6t+9$

$\frac{du}{dt} = 6$

$\frac{du}{6} = dt$

Possibilities:

(a) $21(6x+9)^{21} - 20 \cdot 9^{21}$

(b) $\frac{1}{21}(6x+9)^{21} - \frac{9^{21}}{21}$

(c) $\frac{1}{21}x^{21} - \frac{9^{21}}{21}$

(d) $\frac{1}{6(21)}(6x+9)^{21} - \frac{9^{21}}{6(21)}$

(e) $\frac{1}{20}(6x+9)^{20} - \frac{9^{20}}{20}$

$$\Rightarrow \int_0^x (6t+9)^{20}\, dt = \int_{t=0}^{t=x} u^{20} \cdot \frac{du}{6} = \left.\frac{u^{21}}{21 \cdot 6}\right|_{t=0}^{t=x}$$

$$= \left.\frac{(6t+9)^{21}}{21 \cdot 6}\right|_0^x = \frac{(6x+9)^{21}}{21 \cdot 6} - \frac{(9)^{21}}{21 \cdot 6}$$

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10. A car is traveling due east. Its velocity (in miles per hour) at time t hours is given by $v(t) = -2.4t^2 + 14t + 60$. How far did the car travel during the first 7 hours of the trip?

Possibilities:

(a) 282.8 miles

(b) 19.6 miles

(c) 40.4 miles

(d) 69.8 miles

(e) 488.6 miles

Distance traveled $= \int_0^7 v(t)\, dt$

$$= \int_0^7 (-2.4t^2 + 14t + 60)\, dt = \left. -\frac{2.4t^3}{3} + \frac{14t^2}{2} + 60t \right|_0^7$$

$$= -\frac{2.4(7)^3}{3} + \frac{14(7)^2}{2} + 60(7) - 0 - 0 - 0$$

$$= 488.6$$

11. The graph of $y = f(x)$ shown below consists of straight lines. Evaluate the definite integral $\int_{-3}^{3} f(x)\,dx$.

Possibilities:

(a) 21.5

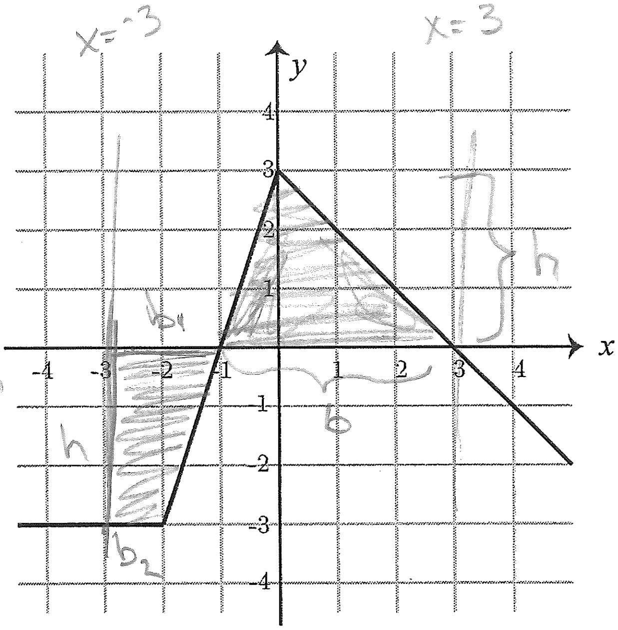
(b) 7.5

(c) 1.5

(d) 6

(e) 2.5

$\int_{-3}^{3} f(x)dx$ is the area between the function and the x-axis between x = -3 and x = 3.



$= -$ Area of trapezoid + Area of triangle

$= -\frac{(b_1+b_2)h}{2} + \frac{b\cdot h}{2} = -\frac{(2+1)(3)}{2} + \frac{4\cdot 3}{2} = -\frac{9}{2} + 6$

$= 1.5$

12. Suppose that $\int_9^{18} f(x)\,dx = 19$ and $\int_5^{18} f(x)\,dx = 8$. Find the value of $\int_5^{9} f(x)\,dx$.

Possibilities:

(a) -27

(b) 27

(c) $-\frac{11}{4}$

(d) -11

(e) 11

$\int_5^{18} f(x)dx = \int_5^{9} f(x)dx + \int_9^{18} f(x)dx$

$\Rightarrow 8 = \int_5^9 f(x)dx + 19$

$\Rightarrow \int_5^9 f(x)dx = 8 - 19 = \underline{\underline{-11}}$

13. Let $f(x) = 9x^2 + 5x + 8$. Find a value c between $x = 2$ and $x = 6$, so that the average rate of change of $f(x)$ from $x = 2$ to $x = 6$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$.

Possibilities:

(a) 2

(b) 3

(c) 4

(d) 5

(e) 6

Average rate of change $= \dfrac{f(6) - f(2)}{6-2}$

$= \dfrac{9(6)^2 + 5(6) + 8 - (9(2)^2 + 5(2) + 8)}{4} = 77$

Instantaneous rate of change $= f'(c)$

$f'(x) = 18x + 5 \Rightarrow$ find when $18c + 5 = 77$

$18c = 72$

$c = 4$

14. For the function

$$f(x) = \begin{cases} |8 + 5x| & \text{if } x < -3 \\ \sqrt{x^2+3} & \text{if } -3 \le x < 4 \\ 2x^2 + 4x + 3 & \text{if } 4 \le x \end{cases}$$

find $\lim_{x \to 6^+} f(x)$

Possibilities:

(a) $\sqrt{19}$

(b) 38

(c) $\sqrt{39}$

(d) 51

(e) 99

$\lim_{x\to 6^+} f(x) = \lim_{x \to 6^+} 2x^2 + 4x + 3$

$= 2(6)^2 + 4(6) + 3$

$= 99$

15. For the function $f(x) = \ln(6x^2 + 5x + 7)$, find the equation of the tangent line to the graph of f at $x = 0$.

Possibilities:

(a) $y = \frac{5}{7}x + \ln(7)$

(b) $y = 7$

(c) $y = \ln(7)\,x + 5$

(d) $y = \frac{7}{5}x + \ln(7)$

(e) $y = \dfrac{(12x+5)\,x}{6x^2+5x+7} + \ln(7)$

equation of tangent line

$= y - f(0) = f'(0)(x-0)$

$f'(x) = \dfrac{1}{6x^2+5x+7} \cdot (12x+5)$

$\Rightarrow f'(0) = \dfrac{(0+5)}{(0+0+7)} = \dfrac{5}{7}$

$f(0) = \ln(0+0+7) \Rightarrow y - \ln(7) = \frac{5}{7}(x-0)$

$= \ln(7)$

$\Rightarrow y = \frac{5}{7}x + \ln(7)$

16. The graph of $y = f(x)$ is shown below. The function is differentiable, except at $x =$

Possibilities:

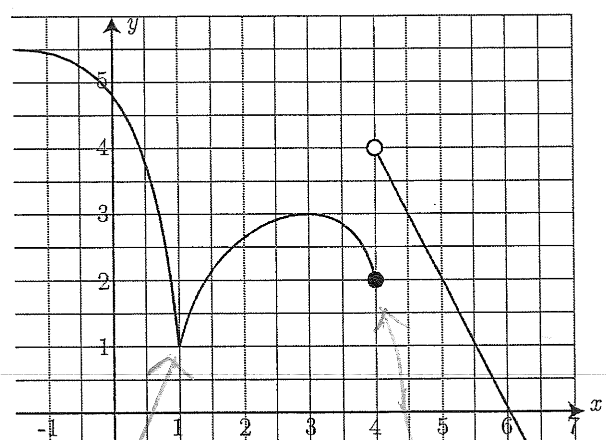
(a) x=4 only

(b) x=1, x=3, x=4, and x=6

(c) x=1 and x=4

(d) x=1 only

(e) x=1, x=3, and x=4



not differentiable at corner points

Not differentiable when f(x) is not continuous

17. If $f(x) = x^7 + 3x^3 + 9x^2$ then find the second derivative $f''(x)$:

Possibilities:

(a) $7x^6 + 9x^2 + 18x$

(b) $42x^5 + 18x + 18$

(c) $42x^5 + 70x^3 + 32x + 18$

(d) $7x^6 + 21x^5 + 35x^4 + 35x^3 + 30x^2 + 34x + 13$

(e) $49x^7 + 27x^3 + 36x^2$

$f'(x) = 7x^6 + 9x^2 + 18x$

$f''(x) = 42x^5 + 18x + 18$

18. Find the derivative, $f'(x)$, if $f(x) = (19x + 11)\, e^{5x+7}$.

Possibilities:

(a) $(19x + 11)(5x + 7)\, e^{5x+6} + 19e^{5x+7}$

(b) $19\,(5x + 7)\, e^{5x+6}$

(c) $19e^5$

(d) $19 \cdot 5e^{5x+7}$

(e) $5(19x + 11)\, e^{5x+7} + 19e^{5x+7}$

product and chain rules

$f'(x) = (19x+11)'\, e^{5x+7} + (19x+11)(e^{5x+7})'$

$= 19e^{5x+7} + (19x+11)\, e^{5x+7}(5)$

19. Suppose $g(-3) = 7$ and $g'(-3) = 10$. Find $F'(-3)$ if

$$F(x) = \frac{g(x)}{x^2}$$

Possibilities:

(a) $-\frac{10}{3}$
(b) $\frac{44}{27}$
(c) $\frac{44}{3}$
(d) $-\frac{44}{27}$
(e) $-\frac{44}{9}$

By the quotient rule,

$$F'(x) = \frac{g'(x)\cdot x^2 - g(x)\cdot 2x}{x^4}$$

$$\Rightarrow F'(-3) = \frac{g'(-3)\cdot(-3)^2 - g(-3)\cdot 2(-3)}{3^4}$$

$$= \frac{10\cdot 9 - 7(-6)}{81} = \frac{132}{81} = \frac{44}{27}$$

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20. Suppose the derivative of $g(t)$ is $g'(t) = 13(t-6)(t-10)$. For t in which interval(s) is g concave up?

Possibilities:

(a) $(8, \infty)$
(b) $(6, 10)$
(c) $(-\infty, 6) \cup (10, \infty)$
(d) $(-\infty, 8)$
(e) $(6, 8) \cup (10, 13)$

g is concave up when $g''(t) > 0$

$g'(t) = 13(t^2 - 16t + 60)$

$\Rightarrow g''(t) = 13(2t - 16)$

$g''(t) = 0$ when $2t - 16 = 0 \Rightarrow t = 8$

check   check $t=0$ check $t=9$

$\cap$ $t=8$ $\cup$

$\Rightarrow$ concave up on $(8, \infty)$

21. A farmer currently has harvested 190 bushels of apples that are currently worth \$13.02 per bushel. The way things are going, he expects to be harvesting 4.00 bushels per day, and expects the price to be increasing at \$0.75 per bushel per day. What is the instantaneous rate of change (measured in dollars per day) of the total value of his apples?

Possibilities:

(a) \$194.56 per day

(b) \$194.57 per day

(c) \$194.58 per day

(d) \$194.59 per day

(e) \$194.60 per day

Total Value = (# of bushels)(Amount per bushel)

$\Rightarrow V(t) = B(t) \cdot A(t)$

All quantities depend on time

$\Rightarrow$ By the product rule $\frac{dV}{dt} = \frac{dB}{dt} \cdot A + B \cdot \frac{dA}{dt}$

$\Rightarrow \frac{dV}{dt} = (4)(13.02) + (190)(0.75)$

$= \boxed{194.58}$

22. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$. Where is the original function $f(x)$ increasing?

Possibilities:

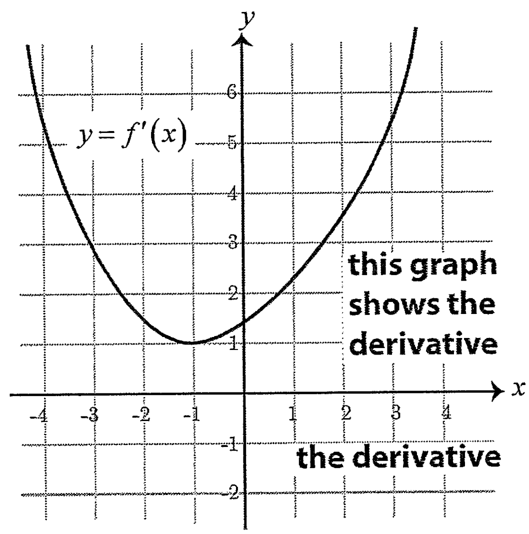
(a) nowhere

(b) $(-1, \infty)$

(c) $(-\infty, -1)$

(d) $(-\infty, \infty)$

(e) $(1, \infty)$



The original function is increasing when $f'(x)$ is positive

From the graph, $f'(x)$ is alwas positive

$\Rightarrow f(x)$ increasing on $(-\infty, \infty)$

Some Formulas

1. **Areas:**

   (a) Triangle $A = \frac{bh}{2}$

   (b) Circle $A = \pi r^2$

   (c) Rectangle $A = lw$

   (d) Trapezoid $A = \frac{h_1 + h_2}{2} b$

2. **Volumes:**

   (a) Rectangular Solid $V = lwh$

   (b) Sphere $V = \frac{4}{3}\pi r^3$

   (c) Cylinder $V = \pi r^2 h$

   (d) Cone $V = \frac{1}{3}\pi r^2 h$