MA123 — Elem. Calculus Fall 2017
Final Exam 2017-12-12

Name: SOLUTIONS Sec.: _____

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(a) b c d e

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GOOD LUCK!

3.	a	**c**	d	e		13.	**a**	b	c	d	e
4.	**a**	b	c	d	e		14.	a	b	c	d	**e**
5.	a	b	c	d	**e**		15.	a	b	c	**d**	e
6.	a	**b**	c	d	e		16.	a	**b**	c	d	e
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9.	a	b	c	**d**	e		19.	a	**b**	c	d	e
10.	**a**	b	c	d	e		20.	a	b	c	d	**e**
11.	a	b	c	d	**e**		21.	a	**b**	c	d	e
12.	a	b	**c**	d	e		22.	a	b	c	**d**	e

For grading use:

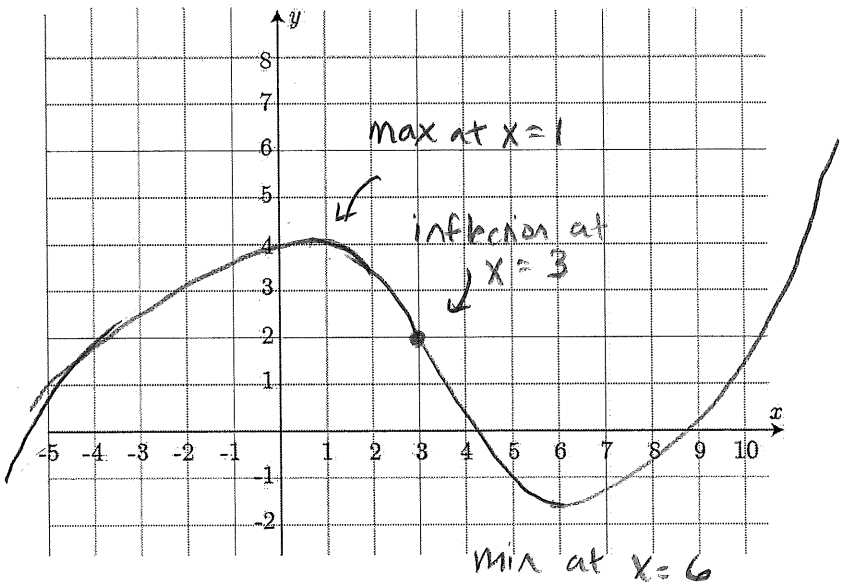
Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(max 110 points)

Fall 2017 Exam 4 Short Answer Questions

Write answers on this page. Your work must be clear and legible to be sure you will get full credit.

1. Sketch the graph of a **continuous** function $y = f(x)$ which satisfies $f'(x) > 0$ on $(-\infty, 1)$ and on $(6, \infty)$, $f'(x) < 0$ on $(1, 6)$; $f''(x) < 0$ on $(-\infty, 3)$ and $f''(x) > 0$ on $(3, \infty)$.



2. Find the average value of the function $f(x) = 4x^3 + 8$ on the interval $[0, 3]$. *You must **clearly show steps using calculus** to find your answer.*

$$\text{Average value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{3-0}\int_0^3 (4x^3 + 8)\,dx$$

$$= \frac{1}{3}\cdot \left. x^4 + 8x \right|_0^3$$

$$= \frac{1}{3}\cdot \left(3^4 + 8(3) - 0 + 0\right)$$

$$= \frac{1}{3}(81 + 24) = \frac{105}{3} = 35$$

Average value: _____

Name: SOLUTIONS

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

3. Suppose you are given the following data points for a function $f(x)$.

x	0	2	4	6	8	10
$f(x)$	4	9	16	19	26	29

Use this data and a **right-endpoint** Riemann sum with five equal subdivisions to estimate the integral, $\int_0^{10} f(x)\,dx$.

Possibilities:

(a) 148

(b) 198

(c) 173

(d) 103

(e) 206

$= 2 \cdot f(2) + 2 \cdot f(4) + 2 \cdot f(6) + 2 \cdot f(8) + 2 \cdot f(10)$

$= 2\,(9 + 16 + 19 + 26 + 29)$

$= 2(99) = 198$

4. Suppose that $\int_4^{19} f(x)\,dx = 195$. Find the average value of $f(x)$ on $[4, 19]$.

Possibilities:

(a) 13

(b) $\frac{195}{2}$

(c) 195

(d) 14

(e) 15

$\frac{1}{19-4} \cdot \int_4^{19} f(x)\,dx$

$= \frac{1}{15}\,(195)$

$= \frac{195}{15} = \frac{39}{3} = 13$

5. Assuming $x > 0$, evaluate the definite integral

$$\int_5^x \frac{5}{t^3}\,dt = \int_5^x 5t^{-3}\,dt$$

Possibilities:

(a) $\frac{5}{\frac{1}{4}x^4} - \frac{4}{125}$

(b) $10\sqrt{x} - 10\sqrt{5}$

(c) $5\ln(|x^3|) - 5\ln(5^3)$

(d) $5\sqrt{x}$

(e) $-\frac{5}{2}(x^{-2}) + \frac{5}{2}(5^{-2})$

$$= \frac{-5}{2}t^{-2}\Big|_5^x$$

$$= \frac{-5}{2}x^{-2} - \left(-\frac{5}{2}\cdot 5^{-2}\right)$$

$$\uparrow \quad \uparrow$$

$$+$$

---

6. Given the function $f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 88 \\ 8x & \text{if } x \geq 88 \end{cases}$

evaluate the definite integral

$$\int_1^{98} f(x)\,dx = \int_1^{88} \frac{1}{x}\,dx + \int_{88}^{98} 8x\,dx$$

Possibilities:

(a) $\frac{654633}{88}$

(b) $\ln(88) + 7440$

(c) 17298

(d) $\ln(88) + 80$

(e) 930

$$= \ln x\Big|_1^{88} + 4x^2\Big|_{88}^{98}$$

$$= \ln 88 - \underbrace{\ln 1}_{0} + 4(98)^2 - 4(88)^2$$

$$= \ln 88 + 7440$$

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7. Let

$$F(x) = \int_0^x \left(t^2 - 9t\right)\ \mathrm{d}t$$

For which positive value of x does $F'(x) = 0$?

Possibilities:

(a) $\frac{9}{2}$

(b) $\frac{27}{2}$

(c) 9

(d) $\frac{477}{2}$

(e) 18

using the fundamental theorem,

$F'(x) = x^2 - 9x$

$F'(x) = 0$ when $x^2 - 9x = 0$

$x(x-9) = 0$

$x = 0$ or $x = 9$

positive

8. Use the Fundamental Theorem of Calculus to compute the derivative, $F'(x)$, of $F(x)$, if

$$F(x) = \int_1^{7x+8} \left(t^2 + 3t + 4\right)\ \mathrm{d}t$$

replace each t with $7x+8$

Possibilities:

(a) $((7x+8)^2 + 3(7x+8) + 4) \cdot (7)$ ← derivative of $7x+8$ (chain rule)

(b) $\frac{1}{3}(7x+8)^3 + \frac{3}{2}(7x+8)^2 + 4(7x+8) - \left(\frac{1}{3}(1)^3 + \frac{3}{2}(1)^2 + 4(1)\right)$

(c) $x^2 + 3x + 4$

(d) $2x + 3$

(e) $(7x+8)^2 + 3(7x+8) + 4$

this would be the answer if we wanted to evaluate the integral, not find $F'(x)$

9. Evaluate the integral

$$\int_0^x (3t+8)^{20}\,dt$$

Possibilities:

(a) $\frac{1}{21}(3x+8)^{21} - \frac{8^{21}}{21}$

(b) $\frac{1}{20}(3x+8)^{20} - \frac{8^{20}}{20}$

(c) $\frac{1}{21}x^{21} - \frac{8^{21}}{21}$

(d) $\frac{1}{3(21)}(3x+8)^{21} - \frac{8^{21}}{3(21)}$

(e) $21(3x+8)^{21} - 20\cdot 8^{21}$

$u = 3t+8$

$\frac{du}{dt} = 3 \qquad \frac{1}{3}du = dt$

[If $t=0$, $u = 3(0)+8 = 8$
If $t=x$, $u = 3x+8$]

$$\int_8^{3x+8} u^{20}\cdot\frac{1}{3}\,du = \frac{1}{3}\cdot\frac{1}{21}u^{21}\Big|_8^{3x+8}$$

$$= \frac{1}{63}(3x+8)^{21} - \frac{1}{63}\cdot 8^{21}$$

10. A car is traveling due east. Its velocity (in miles per hour) at time t hours is given by $v(t) = -2.7t^2 + 16t + 50$. How far did the car travel during the first 4 hours of the trip?

Possibilities:

(a) 270.4 miles

(b) 5.2 miles

(c) 310.4 miles

(d) 77.6 miles

(e) 75.2 miles

(f) 300.8 miles

$$\int_0^4 (-2.7t^2 + 16t + 50)\,dt$$

$$= -\frac{2.7}{3}t^3 + \frac{16}{2}t^2 + 50t\Big|_0^4$$

$$= -.9(4)^3 + 8\cdot 4^2 + 50(4) - 0$$

$$= -57.6 + 128 + 200$$

$$= 270.4 \text{ miles}$$

11. The graph of $y = f(x)$ shown below includes a semicircle and a straight line. Evaluate the definite integral $\int_{-4}^{4} f(x)\,dx$.

Possibilities:

(a) $2\pi + 8$

(b) $-4\pi + 8$

(c) $-2\pi - 8$

(d) $-2\pi + 6$

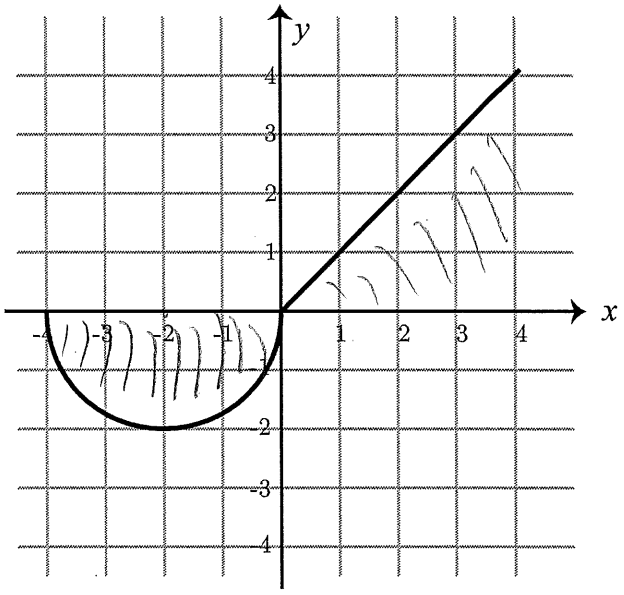
(e) $-2\pi + 8$

half of circle with radius 2:

$\frac{1}{2} \cdot \pi r^2$

$= \frac{1}{2} \cdot \pi \cdot 2^2 = 2\pi$

triangle, base 4 height 4: $\frac{1}{2}(4)(4) = 8$



semicircle below x-axis, triangle above x-axis

12. Suppose that $\int_3^{18} f(x)\,dx = 9$. Find the value of $\int_3^{18} (3f(x) + 30)\,dx$.

Possibilities:

(a) 57

(b) 567

(c) 477

(d) 117

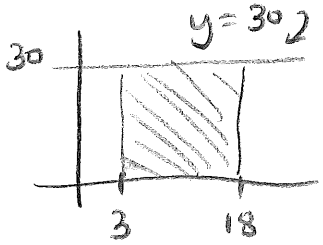
(e) 42

$= 3\int_3^{18} f(x)\,dx + \int_3^{18} 30\,dx$

$= 3(9) + 15(30)$

$= 27 + 450 = 477$

$\int_3^{18} 30\,dx$:



rectangle with base 15, height 30

13. Let $f(x) = x^4$. Find a value c between $x = 0$ and $x = 6$, so that the average rate of change of $f(x)$ from $x = 0$ to $x = 6$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$.

Possibilities:

(a) $\frac{6}{\sqrt[3]{4}}$

(b) 864

(c) $\frac{6}{4}$

(d) 216

(e) $\frac{\sqrt[3]{4}}{6}$

AROC: $\frac{f(6)-f(0)}{6-0} = \frac{6^4-0^4}{6} = 6^3 = 216$

instantaneous rate of change at c

$= f'(c) = 4c^3$

$4c^3 = 216 \Rightarrow c^3 = \frac{216}{4}$ $\quad c = \sqrt[3]{\frac{216}{4}}$

$\Rightarrow c = \frac{\sqrt[3]{216}}{\sqrt[3]{4}} = \frac{6}{\sqrt[3]{4}}$

14. Compute $\lim_{t\to 3} \frac{t^2-9}{t^2+5t-24}$

Possibilities:

(a) 0

(b) The limit does not exist.

(c) 1

(d) $\frac{17}{11}$

(e) $\frac{6}{11}$

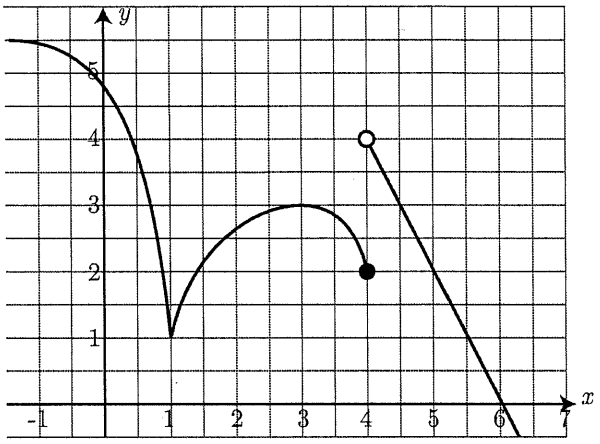
test $t = 3$: $\frac{9-9}{9+15-24} = \frac{0}{0}$

$\lim_{t\to 3} \frac{(t+3)(t-3)}{(t+8)(t-3)}$

$= \lim_{t\to 3} \frac{t+3}{t+8} = \frac{3+3}{3+8}$

$= \frac{6}{11}$

15. The graph of $y = f(x)$ is shown below. The function is **differentiable**, except at $x =$



Possibilities:

(a) x=1, x=3, and x=4

(b) x=1, x=3, x=4, and x=6

(c) x=1 only

(d) x=1 and x=4

(e) x=4 only

"sharp point" at $x = 1$

not continuous at $x = 4$

16. Find the derivative, $f'(x)$, if $f(x) = (17x + 3)\, e^{5x+13}$.

Possibilities:

(a) $17\,(5x + 13)\, e^{5x+12}$

(b) $5\,(17x + 3)\, e^{5x+13} + 17e^{5x+13}$

(c) $17 \cdot 5e^{5x+13}$

(d) $17e^5$

(e) $(17x + 3)\,(5x + 13)\, e^{5x+12} + 17e^{5x+13}$

$f'(x) = (17x+3)e^{5x+13}(5) + e^{5x+13}(17)$

product rule

17. If $f(x) = x^7 + 8x^5 + 3x^4 + 4x^2 + 7$ then find the second derivative $f''(x)$:

Possibilities:

(a) $42x^5 + 160x^3 + 36x^2 + 8$

(b) $49x^7 + 200x^5 + 48x^4 + 16x^2$

(c) $7x^6 + 21x^5 + 75x^4 + 127x^3 + 119x^2 + 67x + 16$

(d) $42x^5 + 230x^3 + 36x^2 + 94x + 14$

(e) $7x^6 + 40x^4 + 12x^3 + 8x$

$f'(x) = 7x^6 + 40x^4 + 12x^3 + 8x + 0$

$f''(x) = 42x^5 + 160x^3 + 36x^2 + 8$

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18. Suppose $g(5) = 8$ and $g'(5) = 6$. Find $F'(5)$ if

$$F(x) = \frac{g(x)}{x^2 - 3}$$ quotient rule

Possibilities:

(a) $\frac{53}{121}$

(b) $\frac{3}{5}$

(c) $\frac{14}{5}$

(d) $\frac{13}{121}$

(e) $\frac{35}{242}$

$$F'(x) = \frac{(x^2-3)\,g'(x) - g(x)(2x)}{(x^2-3)^2}$$

$$F'(5) = \frac{(5^2-3)g'(5) - g(5)(10)}{(5^2-3)^2}$$

$$= \frac{22(6) - 8(10)}{22^2} = \frac{132-80}{484} = \frac{52}{484}$$

$$= \frac{13}{121}$$

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19. Suppose the derivative of $g(t)$ is $g'(t) = 12t^2 - 144t + 324$. For t in which interval(s) is g concave up?

Possibilities:

(a) $(3, 6) \cup (9, 12)$

(b) $(6, \infty)$

(c) $(-\infty, 3) \cup (9, \infty)$

(d) $(-\infty, 6)$

(e) $(3, 9)$

$g''(t) = 24t - 144$

$= 24(t - 6)$

g ∩ ∪

g'' $-$ $+$

test 0 | 6 | test 100

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20. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$.
Where is the original function $f(x)$ increasing?

Possibilities:

(a) $(-\infty, -3)$

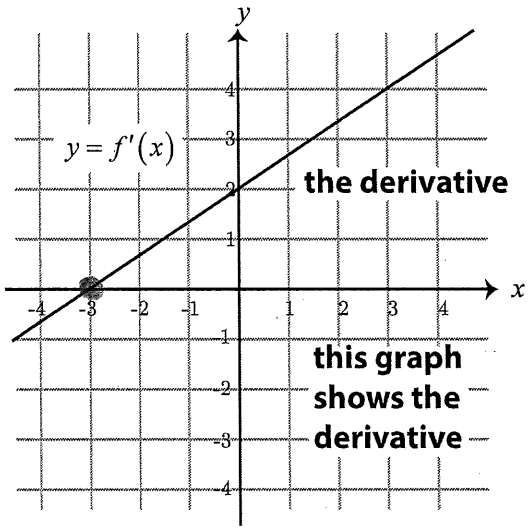
(b) nowhere

(c) everywhere

(d) $(2, \infty)$

(e) $(-3, \infty)$

$f(x)$ is increasing where $f'(x)$ is positive; where the graph of f' is above the x-axis



21. A sphere is growing so its volume is increasing at a rate of 81 cubic feet per minute. At what rate is the radius changing when its radius is 3 feet?

Possibilities:

(a) 2916π feet per minute

(b) $\frac{81}{36\pi}$ feet per minute

(c) $\frac{36\pi}{81}$ feet per minute

(d) $\frac{81}{12\pi}$ feet per minute

(e) $\frac{108\pi}{3}$ feet per minute

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 3 \cdot \frac{4}{3}\pi \cdot r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$81 = 4\pi (3)^2 \frac{dr}{dt}$$

$$81 = 36\pi \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{81}{36\pi}$$

22. Find the area of the largest rectangle whose sides are parallel to the coordinate axes, whose bottom-left corner is at $(0, 0)$ and whose top-right corner is on the graph of $y = 21x - x^2$.

Possibilities:

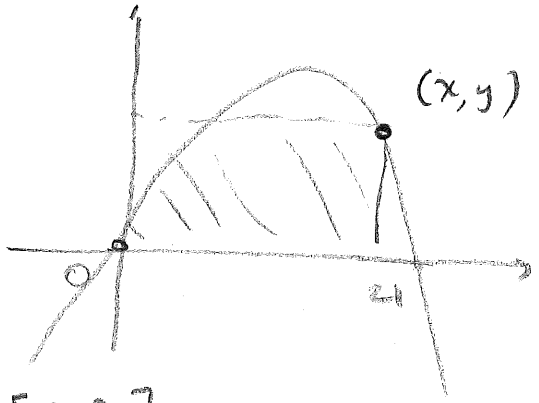
(a) $\frac{21}{2}$

(b) $\frac{9261}{8}$

(c) 0

(d) 1372

(e) 420



$$A = xy$$

$$A = x(21x - x^2)$$

$$A = 21x^2 - x^3$$

$$A' = 42x - 3x^2$$

$A' = 0$ when

$$x(42 - 3x) = 0$$

$x = 0$ or $42 - 3x = 0$ $\quad 3x = 42$ $\quad x = 14$

interval: $[0, 21]$

test: $A(0) = 0$

$A(21) = 0$

$A(14) = 14(21(14) - 14^2)$

$= 14(98) = 1372$

Some Formulas

1. Areas:

(a) Triangle $A = \frac{bh}{2}$

(b) Circle $A = \pi r^2$

(c) Rectangle $A = lw$

(d) Trapezoid $A = \frac{h_1 + h_2}{2} b$

2. Volumes:

(a) Rectangular Solid $V = lwh$

(b) Sphere $V = \frac{4}{3}\pi r^3$

(c) Cylinder $V = \pi r^2 h$

(d) Cone $V = \frac{1}{3}\pi r^2 h$

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