MA123 — Elem. Calculus	Spring 2018	Name: Solutions	Sec.:
Final Exam	2018-5-3		

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and twenty multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name and section number on this page.

GOOD LUCK!

3.	a	b	**c**	d	e	13.	a	**b**	c	d	e
4.	a	**b**	c	d	e	14.	**a**	b	c	d	e
5.	a	b	c	d	**e**	15.	a	b	c	d	**e**
6.	a	b	c	**d**	e	16.	a	b	c	**d**	e
7.	a	b	c	d	**e**	17.	a	b	**c**	d	e
8.	a	b	c	**d**	e	18.	a	**b**	c	d	e
9.	a	b	c	d	**e**	19.	**a**	b	c	d	e
10.	a	**b**	c	d	e	20.	a	b	c	**d**	e
11.	**a**	b	c	d	e	21.	a	b	c	d	**e**
12.	a	b	c	d	**e**	22.	a	b	**c**	d	e

For grading use:

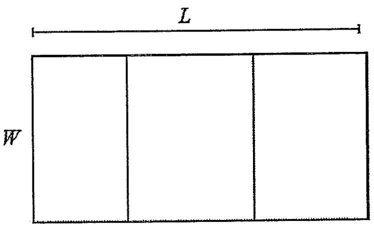
Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(max 110 points)

Spring 2018 Exam 4 Short Answer Questions

Write answers on this page. Your work must be clear and legible to be sure you will get full credit.

1. A farmer builds a rectangular grid of pens with 1 row and 3 columns using 1050 feet of fencing. Find the dimensions (overall length and width) that will maximize the total area of the pen. *You must* ***clearly use calculus*** *to find* ***and justify*** *your answer.*



$2L + 4W = 1050$

$L + 2W = 525$

$L = 525 - 2W$

$A = W \cdot L = W(525 - 2W)$

$= 525W - 2W^2$

$A' = 525 - 4W$

$A' = 0$ when $W = \frac{525}{4}$

$= 131.25$

interval $\left[0, \frac{1050}{4}\right] = [0, 262.5]$

Test $A(0) = 0$, $A\left(\frac{1050}{4}\right) = 0$

$A\left(\frac{525}{4}\right) = \frac{525}{4}\left(\frac{525}{2}\right) = \frac{275625}{8}$

$= 34453.125$

OR

$+ \quad -$

$1 \quad \frac{525}{4} \quad 200$

Width W : $\frac{525}{4} = 131.25$. Overall Length L : $525 - \frac{525}{2} = \frac{525}{2} = 262.5$

2. A truck is traveling due east. Its velocity (in miles per hour) at time t hours is given by $v(t) = -3t^2 + 8t + 80$. How far did the car travel during the first six hours of the trip? (You must ***clearly use calculus*** to find your answer.)

distance $= \int_0^6 (-3t^2 + 8t + 80)\, dt$

$= -t^3 + 4t^2 + 80t \Big|_0^6$

$= -6^3 + 4(6)^2 + 80(6)$

$= -216 + 144 + 480$

$= 408$ miles

Name: Solutions

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

3. Suppose you are given the following data points for a function $f(x)$.

x	0	2	4	6	8	10
$f(x)$	5	8	15	21	27	28

Use this data and a **right-endpoint** Riemann sum with five equal subdivisions to estimate the integral, $\int_0^{10} f(x)\, dx$.

Possibilities:

(a) 152

(b) 104

(c) 198

(d) 175

(e) 208

$$\int_0^{10} f(x)\,dx \approx 8\cdot 2 + 15\cdot 2 + 21\cdot 2 + 27\cdot 2 + 28\cdot 2$$
$$= 16+30+42+54+56$$
$$= 198$$

4. Suppose that the average value of $f(x)$ on $[4, 15]$ is 76. Find the value of $\int_4^{15} f(x)\, dx$.

Possibilities:

(a) 7942

(b) 836

(c) 866

(d) 1672

(e) 418

average value of $f(x)$ on $[4,15]$ $= \frac{1}{15-4}\int_4^{15} f(x)\,dx$

$76 = \frac{1}{11}\int_4^{15} f(x)\,dx$

$836 = 76\cdot 11 = \int_4^{15} f(x)\,dx$

5. Assuming $x > 0$, evaluate the definite integral

$$\int_5^x \frac{17}{t^8}\, dt$$

Possibilities:

(a) $\frac{17}{\frac{1}{7}x^7} - \frac{119}{78125}$

(b) $-\frac{17}{9}(x^{-9}) + \frac{17}{9}(5^{-9})$

(c) $17\ln(|x^8|) - 17\ln(5^8)$

(d) $34\sqrt{x} - 34\sqrt{5}$

(e) $-\frac{17}{7}(x^{-7}) + \frac{17}{7}(5^{-7})$

$$\int_5^x \frac{17}{t^8}\,dt = \int_5^x 17\,t^{-8}\,dt$$

$$= \frac{17\,t^{-7}}{-7}\Big|_5^x = \frac{17x^{-7}}{-7} - \frac{(17)5^{-7}}{-7}$$

6. Given the function $f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 42 \\ 6 & \text{if } x \geq 42 \end{cases}$

evaluate the definite integral

$$\int_1^{52} f(x)\, dx$$

Possibilities:

(a) $\frac{118399}{42}$

(b) $\ln(42) + 2820$

(c) 470

(d) $\ln(42) + 60$

(e) $\ln(42) + 6$

$$= \int_1^{42} f(x)\,dx + \int_{42}^{52} f(x)\,dx$$

$$= \int_1^{42} \frac{1}{x}\,dx + \int_{42}^{52} 6\,dx$$

$$= \ln(x)\Big|_1^{42} + 6\cdot(52-42)$$

$$= \ln(42) - \underbrace{\ln(1)}_{=0} + 60$$

7. Find the value of x at which

$$F(x) = \int_2^x (|t| + 8)\ dt$$

takes its minimum value on the interval $[4, 600]$.

Possibilities:

(a) 12

(b) 2

(c) 600

(d) 184760.0

(e) 4

$F'(x) = |x| + 8$ is always > 0.

So the minimum must be at the left endpoint. ($t > 0$ on $[2, 4]$)

8. Use the Fundamental Theorem of Calculus to compute the derivative, $F'(x)$, of $F(x)$, if

$$F(x) = \int_1^{8x+4} (\ln(t))^3\ dt$$

Possibilities:

(a) $(\ln(x))^3 \cdot (8x + 4)$

(b) $\frac{1}{4}(\ln(8x + 4))^4 \cdot (8)$

(c) $\left(\frac{1}{8x + 4}\right)^3 \cdot (8)$

(d) $(\ln(8x + 4))^3 \cdot (8)$

(e) $(\ln(x))^3 \cdot (8x + 4) \cdot (8)$

$$F'(x) = (\ln(8x+4))^3 \cdot \frac{d}{dx} 8x+4$$
$$= (\ln(8x+4))^3 \cdot 8$$

9. Evaluate the integral

$$\int_0^T 4e^{4x+8}\, dx$$

$u = 4x+8$
$du = 4\,dx$

$= \int_8^{4T+8} e^u\, du$

$= e^u \Big|_8^{4T+8} = e^{4T+8} - e^8$

Possibilities:

(a) $4e^T - 4$

(b) $4e^{4T+8} - 4e^8$

(c) $4e^{4T+8}$

(d) $\frac{4}{9}e^{4T+9}$

(e) $e^{4T+8} - e^8$

10. Suppose a rock is dropped from a Saturnian cliff. After t seconds, its speed in meters per second is $v(t) = 11t$, at least until it lands. If the rock lands after 8 seconds, how high (in meters) is the cliff?

Possibilities:

(a) $\frac{11}{8}$ meters

(b) 352 meters

(c) 4 meters

(d) 8 meters

(e) 88 meters

height $= \int_0^8 11t\, dt$

$= \frac{11t^2}{2}\Big|_0^8 = \frac{11 \cdot 8^2}{2} = 11 \cdot 32 = 352\text{ m}$

11. The graph of $y = f(x)$ shown below consists of straight lines. Evaluate the definite integral

$$\int_{-3}^{3} f(x)\,dx.$$

Possibilities:

(a) 14.5

(b) 17.5

(c) 19

(d) 12

(e) 16

$\int_{-3}^{3} f(x)\,dx$

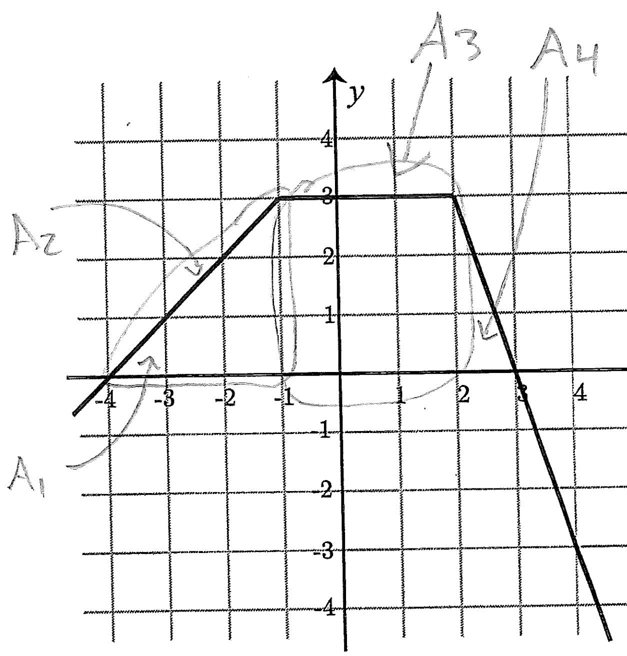
$= -A_1 + A_2 + A_3 + A_4$

$= -\frac{1}{2}(1)(1) + \frac{1}{2}(3)(3)$

$+ 3 \cdot 3 + \frac{1}{2}(3)(1)$

$= -\frac{1}{2} + \frac{9}{2} + 9 + \frac{3}{2}$

$= 14.5$



12. Suppose that $\int_{12}^{16} f(x)\,dx = 27$ and $\int_{3}^{16} f(x)\,dx = 15$. Find the value of $\int_{3}^{12} f(x)\,dx$.

Possibilities:

(a) 42

(b) $-\frac{4}{3}$

(c) 12

(d) -42

(e) -12

$$\underbrace{\int_3^{12} f(x)\,dx}_{\text{want}} + \underbrace{\int_{12}^{16} f(x)\,dx}_{\text{given}} = \underbrace{\int_3^{16} f(x)\,dx}_{\text{given}}$$

$$\int_3^{12} f(x)\,dx + 27 = 15$$

$$\int_3^{12} f(x)\,dx = 15 - 27 = \boxed{-12}$$

13. Find a value of x so that the instantaneous rate of change of $f(x) = 6x^2 + 9$ at x is equal to 12.

Possibilities:

(a) $x = 0$
(b) $x = 1$
(c) $x = 2$
(d) $x = 3$
(e) $x = 4$

inst. rate of change $= f'(x) = 12x$

$12 = 12x$

$1 = x$

14. Find the limit

$$\lim_{t \to 0+} \frac{50\sqrt{t}}{t}$$

Possibilities:

(a) This limit either tends to infinity or this limit fails to exist
(b) 50
(c) 25
(d) $\frac{25}{\sqrt{t}}$
(e) 0

$$\lim_{t \to 0^+} \frac{50\sqrt{t}}{t} = \lim_{t \to 0^+} \frac{50\sqrt{t}}{\sqrt{t}\cdot\sqrt{t}} = \lim_{t \to 0^+} \frac{50}{\sqrt{t}}$$

Plug in $t=0$: $\frac{50}{0}$ $\frac{\text{nonzero}}{\text{zero}}$ so limit does not exist or is infinite

15. The graph of $y = f(x)$ is shown below. $f'(\frac{11}{2})$ is approximately :

Possibilities:

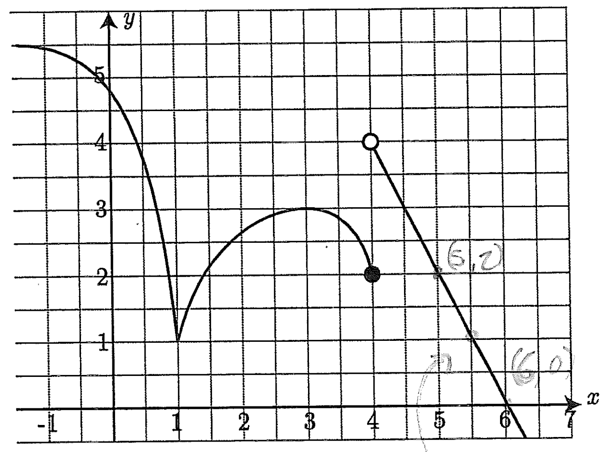
(a) The limit does not exist or tends to infinity

(b) $\frac{1}{2}$

(c) $-\frac{1}{2}$

(d) 2

(e) -2



$f'(\frac{11}{2})$ is slope of this line

$= \frac{0-2}{6-5} = \frac{-2}{1} = -2$

16. Suppose $F(x) = g(x)e^{5x}$. If $g(0) = 3$ and $g'(0) = 7$, find $F'(0)$.

Possibilities:

(a) 7

(b) 15

(c) 35

(d) 22

(e) 10

$F'(x) = g'(x) \cdot e^{5x} + g(x)\, e^{5x} \cdot 5$

$F'(0) = g'(0) \cdot e^{5(0)} + g(0)\, e^{5(0)} \cdot 5$

$= 7 \cdot 1 + 3 \cdot 1 \cdot 5$

$= 7 + 15 = 22$

17. If \$7000 dollars is invested at 6% interest compounded continuously, what is the value of the investment at the end of 3 years?

Possibilities:

(a) \$5846.89

(b) \$8260.00

(c) \$8380.52

(d) \$12600.00

(e) \$42347.53

$P(t) = P_0 e^{.06t} = 7000 e^{.06t}$

$P(3) = 7000 e^{.06(3)} \approx 8380.52$

18. Suppose $g(6) = 5$ and $g'(6) = 4$. Find $F'(6)$ if

$$F(x) = \frac{g(x)}{x^2 - 3}$$

Possibilities:

(a) $\frac{64}{363}$

(b) $\frac{8}{121}$

(c) $\frac{7}{3}$

(d) $\frac{28}{363}$

(e) $\frac{1}{3}$

$$F'(x) = \frac{(x^2-3)g'(x) - g(x) \cdot 2x}{(x^2-3)^2}$$

$$F'(6) = \frac{(6^2-3)g'(6) - g(6) \cdot 2(6)}{(6^2-3)^2}$$

$$= \frac{33 \cdot 4 - 5 \cdot 12}{(33)^2} = \frac{132 - 60}{33^2} = \frac{72}{33^2}$$

$$= \frac{24}{363} = \frac{8}{121}$$

19. Suppose the derivative of $g(t)$ is $g'(t) = 11(t-4)(t-8)$. For t in which interval(s) is g concave up?

Possibilities:

(a) $(6, \infty)$

(b) $(-\infty, 6)$

(c) $(-\infty, 4) \cup (8, \infty)$

(d) $(4, 8)$

(e) $(4, 6) \cup (8, 11)$

$g'(t) = 11(t^2 - 12t + 32)$

$g''(t) = 11(2t - 12) = 22(t-6)$

$g''(t) > 0$ if $t > 6$

20. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$. Where is the original function $f(x)$ increasing?

Possibilities:

(a) $(-2, \infty)$

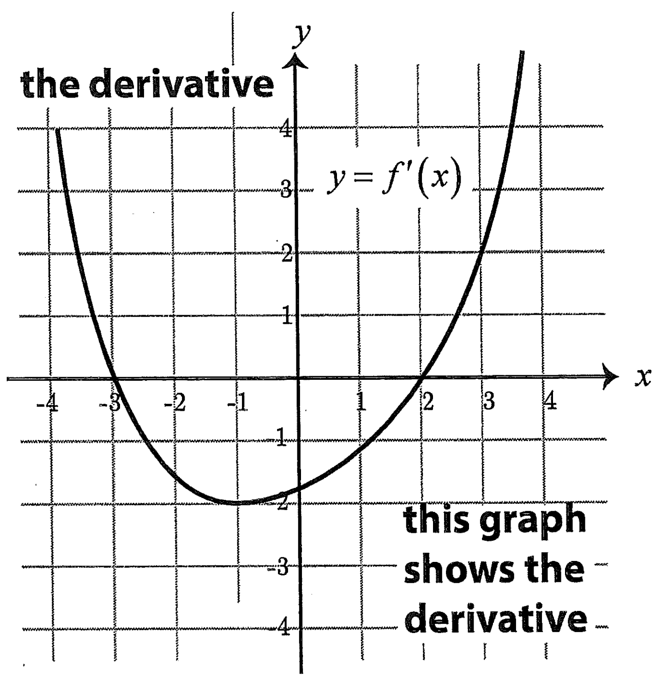
(b) $(-\infty, -1)$

(c) $(-3, 2)$

(d) $(-\infty, -3)$ and $(2, \infty)$

(e) $(-1, \infty)$

f is incr if $f'(x) > 0$



21. A cylindrical water tank with its circular base parallel to the ground is being filled at the rate of 61 cubic feet per minute. The radius of the tank is 5 feet. How fast is the level of the water in the tank rising when the tank is half full?

Possibilities:

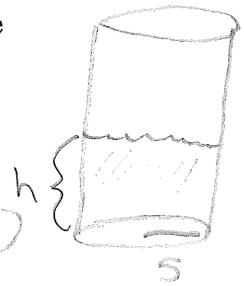
(a) 1525π feet per minute

(b) 5π feet per minute

(c) $\frac{25\pi}{61}$ feet per minute

(d) $\frac{61}{50\pi}$ feet per minute

(e) $\frac{61}{25\pi}$ feet per minute



$\frac{dV}{dt} = 61 \ ft^3/min$

$V = \pi 5^2 \cdot h = 25\pi h$

Want $\frac{dh}{dt}$

But $\underbrace{\frac{dV}{dt}}_{61} = 25\pi \frac{dh}{dt}$, so $\frac{dh}{dt} = \frac{61}{25\pi}$ ft/min

22. A box is constructed out of two different types of metal. The metal for the top and bottom, which are both square, costs \$7 per square foot, and the metal for the four sides costs \$5 per square foot. The box has a volume of 50 cubic feet. If we find the dimensions that minimize cost, what is the length of the base?

Possibilities:

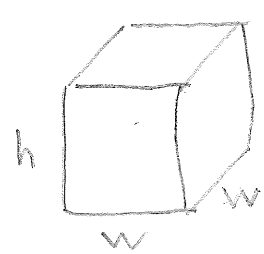
(a) 2.79 feet

(b) 4.29 feet

(c) 3.29 feet

(d) 3.79 feet

(e) 4.79 feet



$V = w^2 \cdot h = 50 \ ft^3 \quad h = \frac{50}{w^2}$

$C = 2 \cdot w^2 \cdot 7 + 4 \cdot h \cdot w \cdot 5$

$= 14w^2 + 20\left(\frac{50}{w^2}\right)w$

$= 14w^2 + \frac{1000}{w}$

$C' = 28w - \frac{1000}{w^2}$

$28w - \frac{1000}{w^2} = 0$

$28w = \frac{1000}{w^2}$

$w^3 = \frac{1000}{28} = \frac{250}{7}$

$\Rightarrow w = \sqrt[3]{\frac{250}{7}} \approx 3.29$

C': $-$, $+$; 1, $\sqrt[3]{50}$, 10

So C has a min at $\sqrt[3]{\frac{250}{7}} \approx 3.29$

Some Formulas

1. **Areas:**
   (a) Triangle $A = \dfrac{bh}{2}$
   (b) Circle $A = \pi r^2$
   (c) Rectangle $A = lw$
   (d) Trapezoid $A = \dfrac{h_1 + h_2}{2} b$
2. **Volumes:**
   (a) Rectangular Solid $V = lwh$
   (b) Sphere $V = \dfrac{4}{3}\pi r^3$
   (c) Cylinder $V = \pi r^2 h$
   (d) Cone $V = \dfrac{1}{3}\pi r^2 h$