MA123 — Elem. Calculus Final Exam	Spring 2019 2019-04-29	**Name:** ____________ **Sec.:** ______

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and twenty multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name and section number on this page.

GOOD LUCK!

3.	a	b	c	d	e	**13.**	a	b	c	d	e
4.	a	b	c	d	e	**14.**	a	b	c	d	e
5.	a	b	c	d	e	**15.**	a	b	c	d	e
6.	a	b	c	d	e	**16.**	a	b	c	d	e
7.	a	b	c	d	e	**17.**	a	b	c	d	e
8.	a	b	c	d	e	**18.**	a	b	c	d	e
9.	a	b	c	d	e	**19.**	a	b	c	d	e
10.	a	b	c	d	e	**20.**	a	b	c	d	e
11.	a	b	c	d	e	**21.**	a	b	c	d	e
12.	a	b	c	d	e	**22.**	a	b	c	d	e

For grading use:

Multiple Choice		**Short Answer**
(number right)	(5 points each)	(out of 10 points)

Total	
	(maximum 110 points)

Spring 2019 Exam 4 Short Answer Questions

Write answers on this page. You must show appropriate legible work to be sure you will get full credit.

1. Let $f(x) = x^2 + 3$. Find a value of x such that the **average rate of change** of $f(x)$ from 1 to x equals 12.

Average rate of change of f(x) from 1 to x

$$= \frac{f(x) - f(1)}{x - 1} = \frac{x^2 + 3 - (1^2 + 3)}{x - 1}$$

$$= \frac{x^2 - 1}{x - 1} = \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = x + 1$$

Set $x + 1 = 12$ solve for x

$$\Rightarrow \underline{\underline{x = 11}}$$

2. Find the **average value** of the function $f(x) = 5x^4 + 10$ on the interval $[0,3]$. *You must* ***clearly show steps using calculus*** *to find your answer.*

Average value of f(x) from 0 to 3

$$= \frac{1}{3-0}\int_0^3 f(x)\,dx = \frac{1}{3}\int_0^3 (5x^4 + 10)\,dx$$

$$= \frac{1}{3}\left[x^5 + 10x \Big|_0^3 \right] = \frac{1}{3}\left[3^5 + 10(3) - 0\right]$$

$$= \frac{1}{3}\left[273\right] = \underline{\underline{91}}$$

Name: ______________________________

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

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3. Suppose you are given the following data points for a function $f(x)$.

x	0	2	4	6	8	10
$f(x)$	5	8	15	21	27	28

Use this data and a $\boxed{\textbf{right-endpoint}}$ Riemann sum with five equal subdivisions to estimate the integral, $\int_0^{10} f(x)\,\mathrm{d}x$.

Possibilities:

(a) 152

(b) 99

(c) 198

(d) 175

(e) 208

$$\int_0^{10} f(x)dx \approx \text{base}(\text{sum of heights})$$
$$= 2(f(2)+f(4)+f(6)+f(8)+f(10))$$
$$= 2(8+15+21+27+28)$$
$$= 198$$

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4. Suppose that the $\underbrace{\text{average value of } f(x) \text{ on } [4,15]}$ is 76. Find the value of $\int_4^{15} f(x)\,\mathrm{d}x$.

Possibilities:

(a) $\dfrac{76}{11}$

(b) 7942

(c) 1672

(d) 418

(e) 836

$$\frac{1}{15-4}\int_4^{15} f(x)dx = 76$$
$$\frac{1}{11}\int_4^{15} f(x)dx = 76$$
$$\int_4^{15} f(x)dx = 76\cdot 11 = 836$$

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5. Evaluate the definite integral

$$\int_5^x 6\sqrt{t}\ \mathrm{d}t = \int_5^x 6t^{1/2}\,dt$$

$$= \frac{6t^{3/2}}{3/2}\Big|_5^x = 4t^{3/2}\Big|_5^x$$

$$= 4x^{3/2} - 4(5)^{3/2}$$

Possibilities:

(a) $12\sqrt{x} - 12\sqrt{5}$

(b) $6x^{\frac{3}{2}} - 6 \cdot 5^{\frac{3}{2}}$

(c) $6\sqrt{x}$

(d) $4x^{\frac{3}{2}} - 4 \cdot 5^{\frac{3}{2}}$

(e) $\frac{6}{\sqrt{x}} - \frac{6}{\sqrt{5}}$

6. Given the function $f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 42 \\ 6x & \text{if } x \geq 42 \end{cases}$

evaluate the definite integral

$$\int_1^{52} f(x)\ \mathrm{d}x = \int_1^{42} f(x)dx + \int_{42}^{52} f(x)dx$$

$$= \int_1^{42} \frac{1}{x}\,dx + \int_{42}^{52} 6x\,dx$$

$$= \ln x\Big|_1^{42} + 3x^2\Big|_{42}^{52}$$

$$= \ln(42) - \ln(1) + 3(52)^2 - 3(42)^2$$

$$= \ln(42) + 2820$$

Possibilities:

(a) $\ln(42) + 2820$

(b) $\ln(42) + 60$

(c) 4418

(d) 470

(e) $\frac{118399}{42}$

7. If an amount of x dollars is invested at 3% interest compounded continuously, and at the end of 4 years the value of the investment is \$4000, find x.

Possibilities:

(a) \$3732.28

(b) \$4509.99

(c) \$3547.68

(d) \$3214.27

(e) \$3137.82

$P = P_0 e^{rt}$

$4000 = x e^{(.03)(4)}$

$4000 = e^{.12} \cdot x$

$\frac{4000}{e^{.12}} = x$

$3547.68 \approx x$

8. Use the Fundamental Theorem of Calculus to compute the derivative, $F'(x)$, of $F(x)$, if

$$F(x) = \int_1^{6x+4} \left(t^2 + 8t + 2\right)\, dt$$

Possibilities:

(a) $\frac{1}{3}(6x+4)^3 + \frac{8}{2}(6x+4)^2 + 2(6x+4) - \left(\frac{1}{3}(1)^3 + \frac{8}{2}(1)^2 + 2(1)\right)$

(b) $2x + 8$

(c) $(6x+4)^2 + 8(6x+4) + 2$

(d) $((6x+4)^2 + 8(6x+4) + 2) \cdot (6)$

(e) $x^2 + 8x + 2$

If $F(x) = \int_a^{f(x)} g(t)\,dt$,

then $F'(x) = g(f(x)) \cdot f'(x)$

$\Rightarrow F'(x) = \left[(6x+4)^2 + 8(6x+4) + 2\right] \cdot 6$

9. Evaluate the integral

$$\int_0^x (8t+4)^{10}\ dt$$

Let $u = 8t+4 \Rightarrow \frac{du}{dt} = 8$

$\frac{du}{8} = dt$

$t=0 \Rightarrow u = 4$

$t=x \Rightarrow u = 8x+4$

Possibilities:

(a) $\frac{1}{10}(8x+4)^{10} - \frac{4^{10}}{10}$

(b) $\frac{1}{11}(8x+4)^{11} - \frac{4^{11}}{11}$

(c) $\frac{1}{11}x^{11} - \frac{4^{11}}{11}$

(d) $11(8x+4)^{11} - 10 \cdot 4^{11}$

(e) $\frac{1}{8(11)}(8x+4)^{11} - \frac{4^{11}}{8(11)}$

$$= \int_4^{8x+4} u^{10} \cdot \frac{du}{8} = \frac{u^{11}}{8(11)}\Bigg|_4^{8x+4}$$

$$= \frac{(8x+4)^{11}}{8 \cdot 11} - \frac{4^{11}}{8 \cdot 11}$$

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10. Suppose a rock is dropped from a Saturnian cliff. After t seconds, its speed in meters per second is $v(t) = 11t$, at least until it lands. If the rock lands after 8 seconds, how high (in meters) is the cliff?

Possibilities:

(a) $\frac{11}{8}$ meters

(b) 88 meters

(c) 352 meters

(d) 8 meters

(e) 4 meters

$$\text{Height} = \int_0^8 v(t)\,dt = \int_0^8 11t\,dt$$

$$= \frac{11}{2}t^2\Big|_0^8 = \frac{11}{2}\cdot 8^2 - \frac{11}{2}\cdot 0^2$$

$$= \frac{11}{2}(64)$$

$$= 352$$

11. The graph of $y = f(x)$ shown below consists of straight lines. Evaluate the definite integral $\int_{-3}^{3} f(x)\,\mathrm{d}x$.

$$\int_{-3}^{3} f(x)\,dx = \int_{-3}^{-1} f(x)\,dx + \int_{-1}^{3} f(x)\,dx$$

$$= -\frac{9}{2} + 6$$

$$= \frac{3}{2}$$

Possibilities:

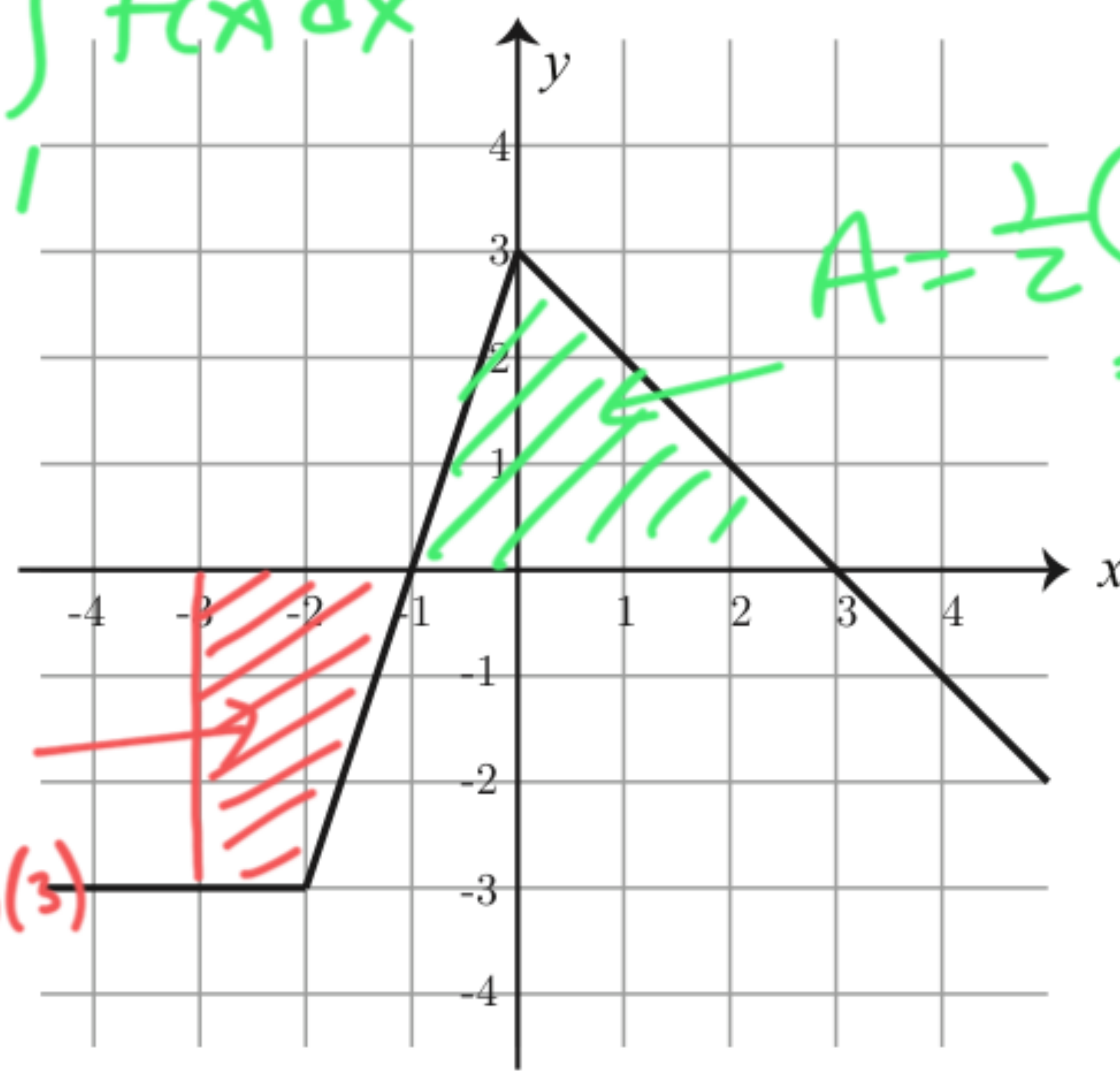
(a) 2.5

(b) 1.5

(c) 7.5

(d) 21.5

(e) 6



$A = \frac{1}{2}(4)(3) = 6$

Area $= \frac{1}{2}(2+1)(3) = \frac{9}{2}$

12. Suppose that $\int_{3}^{12} f(x)\,\mathrm{d}x = 27$, $\int_{16}^{36} f(x)\,\mathrm{d}x = 50$, and $\int_{3}^{36} f(x)\,\mathrm{d}x = 15$. Find the value of $\int_{12}^{16} f(x)\,\mathrm{d}x$.

Possibilities:

(a) 92

(b) −748

(c) −62

(d) 8

(e) −92

$$\int_{3}^{36} f(x)\,dx = \int_{3}^{12} f(x)\,dx + \int_{12}^{16} f(x)\,dx + \int_{16}^{36} f(x)\,dx$$

$$15 = 27 + \int_{12}^{16} f(x)\,dx + 50$$

$$15 - 27 - 50 = \int_{12}^{16} f(x)\,dx$$

$$\boxed{-62 = \int_{12}^{16} f(x)\,dx}$$

13. For the function $f(x) = \ln(x^2 + 9x + 11)$, find the equation of the tangent line to the graph of f at $x = 0$.

$y - y_1 = m(x - x_1)$

$f(0)$ $f'(0)$

Possibilities:

(a) $y = \frac{9}{11}x + \ln(11)$

(b) $y = 11$

(c) $y = \frac{11}{9}x + \ln(11)$

(d) $y = \dfrac{2x^2 + 9x}{x^2 + 9x + 11} + \ln(11)$

(e) $y = \ln(11)\,x + 9$

$f(0) = \ln(0^2 + 9(0) + 11) = \ln(11)$

$f'(x) = \dfrac{2x+9}{x^2+9x+11} \Rightarrow f'(0) = \dfrac{9}{11}$

$\Rightarrow y - \ln(11) = \dfrac{9}{11}(x - 0) \Rightarrow y = \dfrac{9}{11}x + \ln(11)$

14. For the function

$$f(x) = \begin{cases} 6x^2 + 4x + 7 & \text{if } x < -1 \\ \sqrt{x^2 + 1} & \text{if } -1 \leq x < 4 \\ 3\,|2 + 3x| & \text{if } 4 \leq x \end{cases}$$

find $\lim_{x \to -5^+} f(x)$ $= \lim_{x \to -5^+} 6x^2 + 4x + 7$

Possibilities:

(a) $\sqrt{26}$

(b) 119

(c) $\sqrt{17}$

(d) 39

(e) 137

$= 6(-5)^2 + 4(-5) + 7$

$= 137$

15. Consider the graph of the original function, $f(x)$.
For this function, what are the signs of $f'(-3)$ and $f''(-3)$?

$f'(-3)$ = slope of tangent line

$\Rightarrow f'(-3) > 0$

Possibilities:

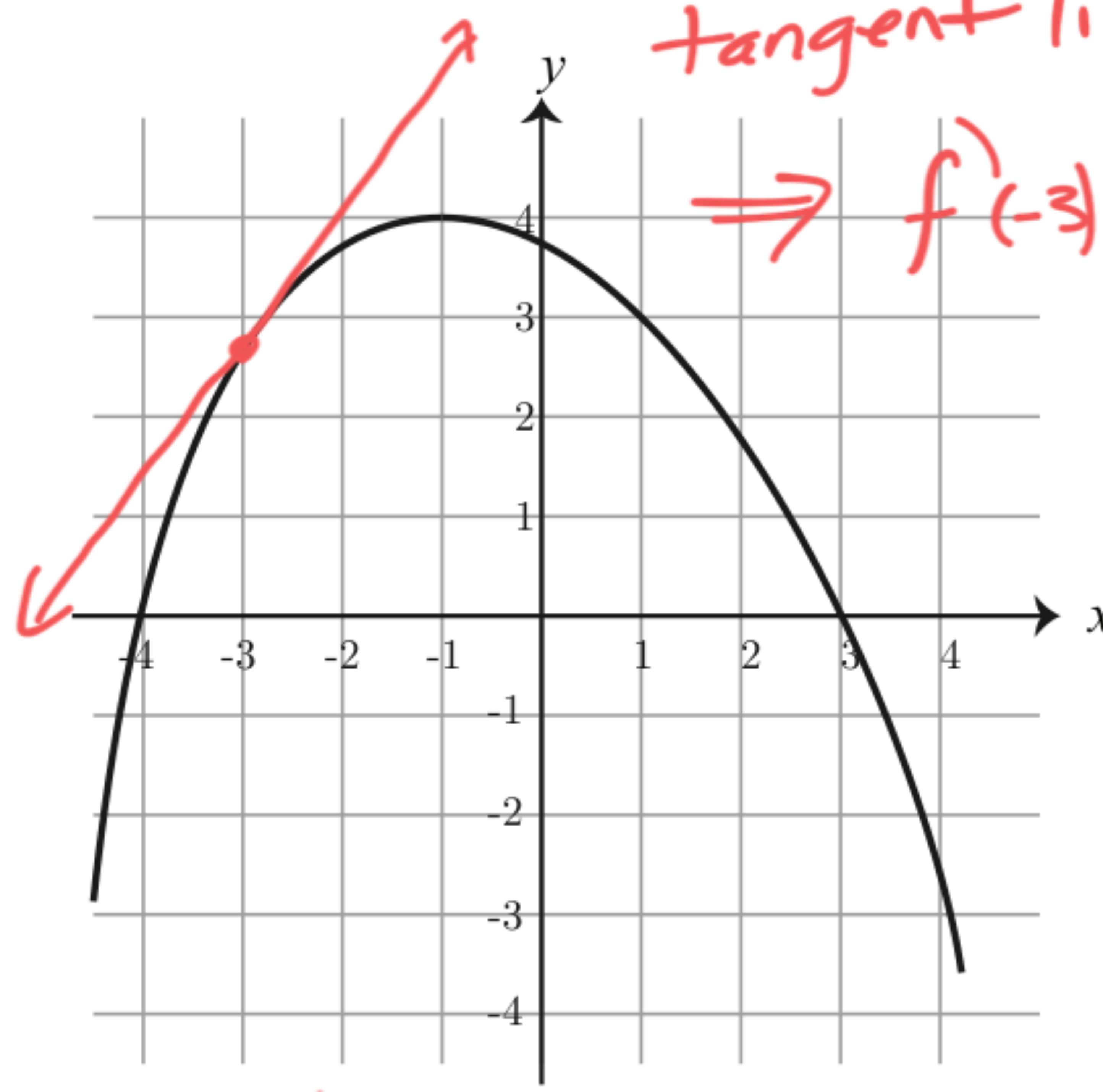
(a) $f'(-3) = 0$ and $f''(-3) < 0$

(b) $f'(-3) > 0$ and $f''(-3) > 0$

(c) $f'(-3) > 0$ and $f''(-3) < 0$

(d) $f'(-3) < 0$ and $f''(-3) < 0$

(e) $f'(-3) < 0$ and $f''(-3) > 0$



At $x = -3$ $f(x)$ is concave down

$\Rightarrow f''(-3) < 0$

16. Suppose $F(x) = (x + 5)e^{g(x)}$. If $g(9) = 0$, and $g'(9) = 3$, find $F'(9)$.

Product rule

Possibilities:

(a) 3

(b) 42

(c) 43

(d) 15

(e) 0

$$F'(x) = (x+5)'e^{g(x)} + (x+5)\left[e^{g(x)}\right]'$$

$$= 1 \cdot e^{g(x)} + (x+5)e^{g(x)} \cdot g'(x)$$

$$\Rightarrow F'(9) = e^{g(9)} + (9+5)e^{g(9)} \cdot g'(9)$$

$$= e^0 + 14e^0 \cdot 3 = 1 + 42 = \boxed{43}$$

17. The total cost (in dollars) of producing x machines is

$$C(x) = 2500 + 30x - .1x^2.$$

Use the **marginal cost** to approximate the cost of producing the 21st machine.

Possibilities:

(a) \$26.00

(b) \$25.90

(c) \$3085.90

(d) \$28.00

(e) \$146.95

Cost of 21^{st} machine

$\approx C'(20)$

$C'(x) = 30 - .2x$

$C'(20) = 30 - .2(20) = 26$

18. Suppose $g(6) = 5$ and $g'(6) = 4$. Find $F'(6)$ if

$$F(x) = \frac{x^2+1}{g(x)}$$

quotient rule

Possibilities:

(a) $-\frac{84}{5}$

(b) $-\frac{7}{3}$

(c) 3

(d) $\frac{84}{25}$

(e) $-\frac{88}{25}$

$$F'(x) = \frac{(x^2+1)'g(x) - (x^2+1)g'(x)}{(g(x))^2}$$

$$\Rightarrow F'(x) = \frac{2xg(x) - (x^2+1)g'(x)}{(g(x))^2}$$

$$\Rightarrow F'(6) = \frac{2(6)g(6) - (6^2+1)g'(6)}{(g(6))^2}$$

$$= \frac{12(5) - (37)(4)}{5^2} = \boxed{-\frac{88}{25}}$$

19. Where is the function $f(t) = t^3 - 6t^2 - 63t + 8$ concave down?

Find where $f''(t) < 0$

Possibilities:

(a) $f(t)$ is always concave down

(b) $-3 < t < 7$

(c) $t < 2$

(d) $t > 2$

(e) $t < -3$ and $t > 7$

$f'(t) = 3t^2 - 12t - 63$

$f''(t) = 6t - 12 \overset{?}{=} 0$

$\Rightarrow t = 2$

$f''(1) < 0$ $\qquad$ $f''(3) > 0$

2

$\Rightarrow$ Concave down on $(-\infty, 2)$

$t < 2$

20. The following is the graph of ~~the derivative~~, $f'(x)$, of the function $f(x)$.
Where is the regular function $\boxed{f(x) \text{ increasing?}}$

$f'(x) > 0$

Possibilities:

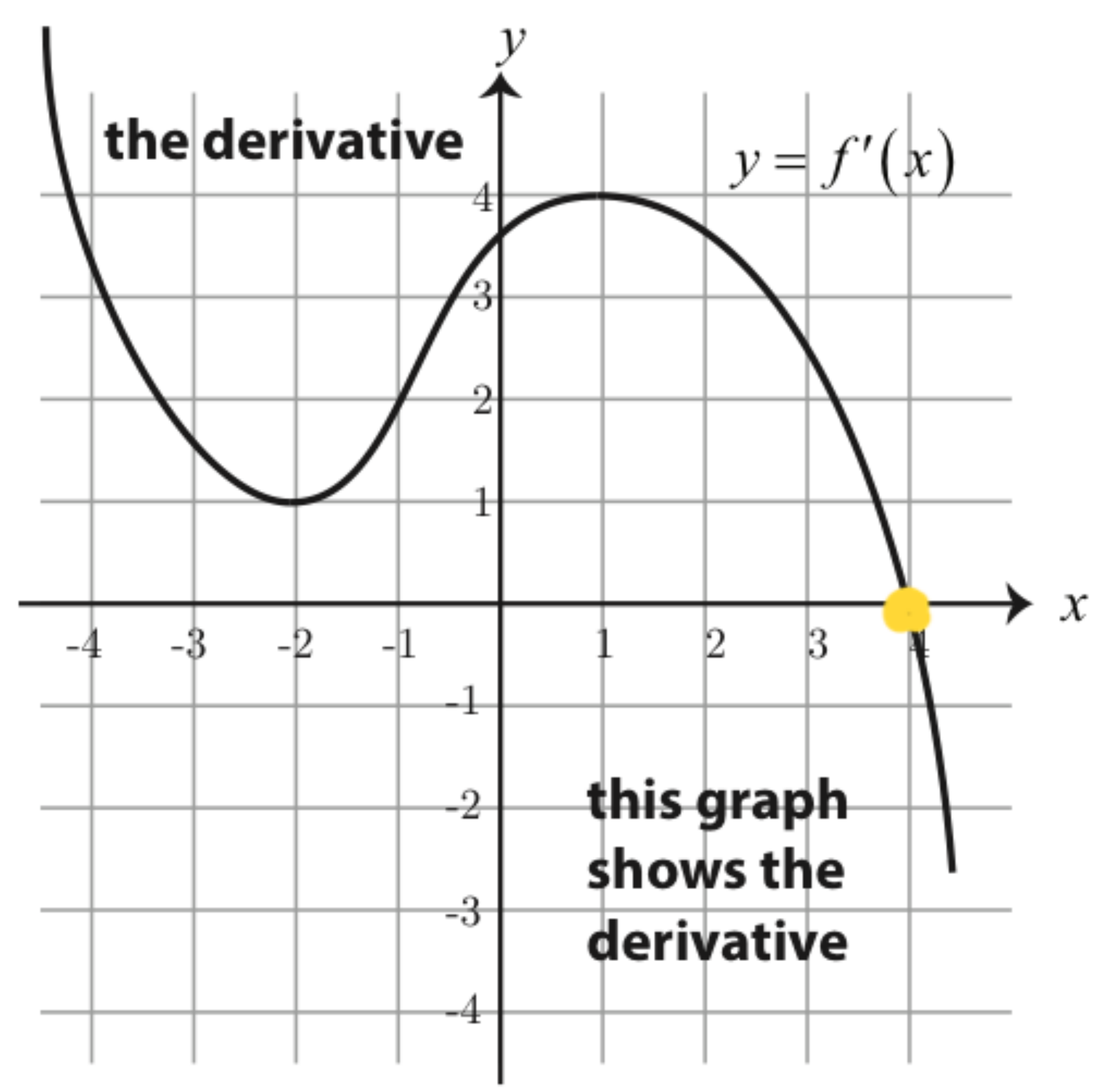
(a) $(-\infty, -1)$

(b) $(-\infty, -2)$ and $(1, \infty)$

(c) $(-2, 1)$

(d) $(4, \infty)$

(e) $(-\infty, 4)$



when $x < 4$, $f'(x) > 0$

$\Rightarrow f(x)$ increasing on $(-\infty, 4)$

21. If a tank holds 500 gallons of water, which drains from the bottom of the tank in 90 minutes, then Torricelli's Law give the volume V of water remaining in the tank after t minutes as

$$V = 500\left(1 - \frac{t}{90}\right)^2.$$

Find the rate at which water is draining out of the tank after 10 minutes. $= V'(10)$

Possibilities:

(a) $\frac{400}{81}$ gallons per minute
(b) $\frac{800}{81}$ gallons per minute
(c) $\frac{100}{9}$ gallons per minute
(d) $\frac{32000}{81}$ gallons per minute
(e) $\frac{8000}{9}$ gallons per minute

$$V' = 500 \cdot 2\left(1 - \frac{t}{90}\right)\cdot\left(-\frac{1}{90}\right)$$

$$= -\frac{100}{9}\left(1 - \frac{t}{90}\right)$$

$$\Rightarrow V'(10) = -\frac{100}{9}\left(1 - \frac{10}{90}\right)$$

$$= -\frac{100}{9}\left(\frac{8}{9}\right) = -\frac{800}{81}$$

draining out

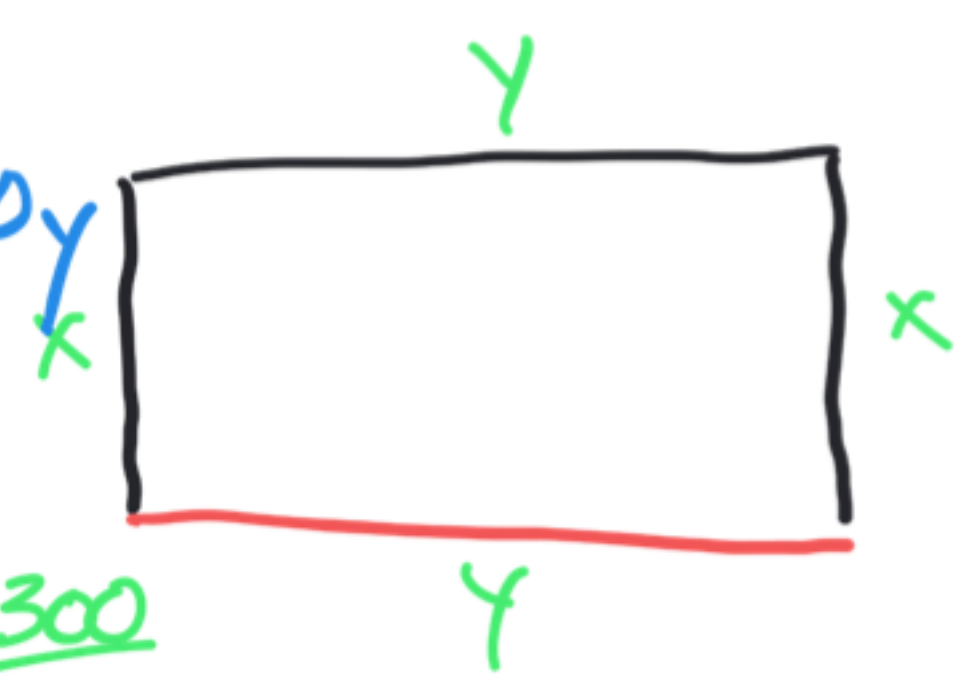
22. A landscape architect wishes to enclose a rectangular garden on one side by a brick wall costing \$50 per foot, and on the other three sides by a metal fence costing \$30 per foot. If the area of the garden is 300 square feet, find the lowest possible cost to enclose the garden.

Possibilities:

(a) \$2399.50
(b) \$2401.00
(c) \$2400.50
(d) \$2400.00
(e) \$2401.50

$$\text{Cost} = 30(2x+y) + 50y$$

$$= 60x + 80y$$



$$x \cdot y = 300 \Rightarrow y = \frac{300}{x}$$

$$\Rightarrow C = 60x + 80(300x^{-1})$$

$$\Rightarrow C' = 60 - 24000x^{-2} = 0$$

$$60 = \frac{24000}{x^2} \Rightarrow x^2 = \frac{24000}{60}$$

$$x = 20$$

$$\Rightarrow C(20) = 60(20) + \frac{24000}{20}$$

$$= 1200 + 1200$$

$$= 2400$$

20

Some Formulas

1. **Areas:**
   (a) Triangle $A = \frac{bh}{2}$
   (b) Circle $A = \pi r^2$
   (c) Rectangle $A = lw$
   (d) Trapezoid $A = \frac{h_1 + h_2}{2} b$

2. **Volumes:**
   (a) Rectangular Solid $V = lwh$
   (b) Sphere $V = \frac{4}{3}\pi r^3$
   (c) Cylinder $V = \pi r^2 h$
   (d) Cone $V = \frac{1}{3}\pi r^2 h$