1 Tiling with the simplex algorithm

Suppose we have two tiles, f and g, and wish to tile a region F. This is equivalent to finding two polynomials A and B such that $F = Af + Bg$ where the coefficients of A and B are all 1.

```
restart;
rect := proc(n,m)
local i,j;
expand(
sum(x^'i','i' = 0 .. m-1)
* sum(y^'j','j' = 0 .. n-1))
end:
```

Now we need some tiles

```
tile1:=1+x+x^2+y:
tile2:=y+x*y+x^2*y+x^2 :
```

Now choose a polynomial region f.

```
f:= rect(4,4);
```

We need a word to convert f to a linear form.

```
cnv := proc(f)
local tmp,tf,i,j;
tmp :=NULL;
tf := collect(simplify(f),[x,y]);
for i from 0 to degree(tf,x) do
for j from 0 to degree(tf,y) do
if not coeff(coeff(tf,x,i),y,j)=0
then
tmp :=
tmp,coeff(coeff(tf,x,i),y,j)*z[i,j];
fi
od
od;
convert([tmp],`+`);
end:
cnv(f);
```

This word will be used to draw the tiling.

```
squares := proc(f)
local tmp,tf,i,j;
tmp :=NULL;
tf := collect(simplify(f),[x,y]);
for i from 0 to degree(tf,x) do
for j from 0 to degree(tf,y) do
if 0 < coeff(coeff(tf,x,i),y,j)
then
tmp := tmp,
[[i+.1,j+.1],[i+1-.1,j+.1],[i+1-.1,j+1-.1],[i+.1,j+1-.1]];
fi
```

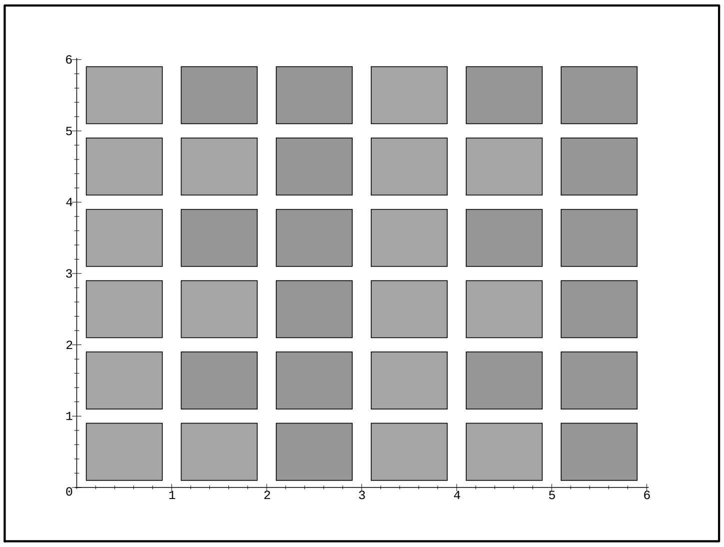
```
od
od;
[tmp]end:
```

Here is the word to search for a solution using the simplex algorithm. The idea is simple:

```
tileit := proc(f,tile1,tile2)
local A, B,i,a,b, gf, p,eqns, sol, sln1, sln2, drawing ;
A := convert([seq(a[i-1]*op(i,f),i=1..nops(f))],`+`);
B := convert([seq(b[i-1]*op(i,f),i=1..nops(f))],`+`);
gf := cnv(expand(A*tile1+tile2*B -f))
;
eqns:=seq(op(1,op(i,gf))=0 ,i=1..nops(gf));
sol:=simplex[maximize](1,[eqns],NONNEGATIVE);
```

```
if not sol={} then sln1 := expand(subs(sol,A)) sln2 := expand(subs(sol,B)) drawing
:= NULL if not sln1 = 0 then
```

```
drawing :=
drawing,seq(plots[polygonplot](squares(expand(
tile1*p)),color=tan) ,p=[op(sln1 )]) fi;
if not sln2 = 0 then
drawing:= drawing,
seq(plots[polygonplot](squares(expand(
tile2*p)),color=green) ,p=[op(sln2)]) fi;
print(plots[display]([drawing]));
else RETURN(`no tiling`) fi;
[sln1,sln2];
end:
f :=rect(7,7)-x^3*y^2-y^4-x^6*y;
tile1 := 1+x+x^2:  tile2 := 1+y+y^2:
tileit(rect(6,6),1+x+y,y+x*y+x);
```



$$[y^4 + x^3 + x^3\,y^2 + 1 + x^3\,y^4 + y^2,\ x\,y^4 + x^4 + x + x^4\,y^2 + x^4\,y^4 + x\,y^2]$$

```
tileit(rect(8,8)-rect(4,4)+rect(2,3)+y^2+x*y^2,rect(1,2) , rect(1,3)
);
tileit(rect(6,6)-x*y-x^3*y^3-x^5*y^5,els(2)[1
],1+x);
undebug(cnv);
els(3);
```