```
> speed:=abs(vel);
```

$$speed := |vel|$$

```
> 5.2#14;
```

Syntax error, unexpected number

```
> 5.1:  7,24,34;
> 5.3:  1,2,5
> p 13:  12,14,16
> p.46:3,8,14
> 5.5:  1,3,5,7
```

REMARKS ON HOMEWORK

Velocity and Speed

Velocity is a quantity with magnitude and direction. Speed is the magnitude of velocity. If a particle is in motion along a straight line then the line may be thought of as a number line, the position along the line at time t, s(t), a function of time and the velocity d/dt s(t) = v(t) a number which may be positive or negative. Positive indicates motion in the positive direction along the line, negative in the opposite direction. The speed is the magnitude of the velocity - in this case its absolute value. If v(t) is the velocity at time t and s(t) is the position at time t then the NET DISPLACEMENT **over the interval [a,b] is s(b)-s(a). By by FTC**

```
> s(b)-s(a) = Int(v(t),t=a..b);
```

$$\mathrm{s}(b) - \mathrm{s}(a) = \int_a^b \mathrm{v}(t)\, dt$$

Note that it is possible for the net displacement to be zero - this is what happens if one starts at time a at s(a) and ends up in the same place at time b, s(a) - s(b). Thederivative of the total displacement (total distance traveled) is the speed —v(t)—. The total distance traveled between time t=a and time t=b is thus

```
> TOTAL_DISTANCE_TRAVELED = Int(abs(v(t)),t=a..b);
```

$$TOTAL\_DISTANCE\_TRAVELED = \int_a^b |\mathrm{v}(t)|\, dt$$

In problem 7 page 12 the velocity v(t) is 60-20t thus the net distance traveled between time t=0 and time t=4 is

```
> Net_distance_traveled:=Int( 60-20*t ,t=0..4);evalf(");
```

$$Net\_distance\_traveled := \int_0^4 60 - 20\,t\,dt$$

$$80.$$

```
> Total_distance_traveled:=Int(abs(60-20*t),t=0 ..4);evalf(");
```

$$Total\_distance\_traveled := \int_0^4 |60 - 20\,t|\,dt$$

$$100.$$

Page 25 number 14

In this problem the functions G and H have the same derivative, hence they differ by a constant. In fact we obviously have the equation

```
> G(x) = Int(f(t),t=-3..x)+Int(f(t),t=-3..2);
```

$$\mathrm{G}(x) = \int_{-3}^{x} \mathrm{f}(t)\,dt + \int_{-3}^{2} \mathrm{f}(t)\,dt$$

So the constant term is the integral of f from -3 to 2 - which we can calculate directly from the graph - its 1/2 - 2Pi.

The question on which intervals H is increasing can be read from the graph of f. A function is increasing when its derivative is positive,

thus H is increasing when f is positive. That we can read off the graph. G has a local minimum at x=1 because its deriviative changes sign from negative to positive at x=1. Similarly, G has a local max at x=-3. Note that G(-3) = 0 and G(1) = -2Pi.. In the interval [-5,-2] G has three critical values: the two end points and the critical number at t=-3, where it has a local max since the derivative changes from + to -.. At the end point

```
> G(-5):=Int(f(t),t=-3..-5);
```

$$\mathrm{G}(-5) := \int_{-3}^{-5} \mathrm{f}(t)\,dt$$

Looking at the graph we see that the integral from -5 to -3 is 3/4 so G(-5) = -3/4. Similarly the vaue at the right endpoint is

```
> Int(-sqrt(4-(x+1)^2),x=-3..-2);
```

$$\int_{-3}^{-2} -\sqrt{3 - x^2 - 2\,x}\,dx$$

We can see that the area this represents inside the semicircle is smaller than the ares of the triangle with vertices (-5,0),(-5,3), (-3,0) so the minimum value value of

-3/4 occurs at the left end point x=-5, and the maximum, 0 , occurs at the critical number x=-3.

H has critical numbers at x=-5, x=-3, x=1, and x=5. Adding areas we see that H has a value of -1/2 at 1, and 2Pi-1/2 at x=-3. The value at x=5 is 3-1/2 (the area of the rightmost triangle - the area from 1 to 2), and the value at x=-5 is 2Pi-1/2- 3. Thus the maximum is 2Pi-1/2 at x=-3 and the minimum is -1/2 at x=1.

Page 35 number 1

F is a function whose derivative, f is positive and is decreasing from 0 to 2 and increasing from 2 to 5. Thus F must be increasing and must go from concave down to concave up as it passes 2. Only A has this property.

Page 35 number 2

This is very similar to many other problems in these sections. What one needs to realize is that one is looking at the graph of the derivative of a function, F,

Adding areas we se that F(0) = 0, F(2) = 4, f(-1) = -3/2. The fact that F(-1) = -3/2 is caused by the fact that

```
> F(-1) = Int(f(x),x=0..-1);
```

$$\mathrm{F}(-1) = \int_0^{-1} \mathrm{f}(x)\,dx$$

The integration going backwards results in the sign being negative. The critical points of F are where its derivitaive,f, is zero or not defined, and the endpoints.

Thus the criticl values of F are 0,-4.5, -3,2,5. To find the max and min we could simply determine the values of F at these points by calculating the areas - remembering that integration from right to left changes the sign. The inflection points of F are where its derivative changes from increasing to decreasing or decreasing to increasing. This F has inflection points at -3,1, and 3.

page 36, problem 4

The graphs of G and g differ by a constant since they have the same derivative. G (5) is the area from 0 to 5 which is a little larger than 6 while g(5)=1,

so G(x) = g(x) + c where c is about. Thus g(0) is about -5. This means that the sign of g changes from - to + between 0 and 5. It changes from + to -

in going from 5 to 8, say because

```
> g(8) = g(1)+Int(Diff(g(x),x),x=1..8);
```

$$\mathrm{g}(8) = \mathrm{g}(1) + \int_1^{8} \frac{\partial}{\partial x}\,\mathrm{g}(x)\,dx$$

And the integral from 6 to 8 is less than -1. There are no other sign changes since the function g decreases until x=9 where its value is less than -2 (comparing the blocks) thus although it increases then by the area from 9 to 10 it can't go up enough to become positive.

page 47 number 12

Correction: The sum in part b is not a Riemann sum approximation to

```
> Int(x^2,x=0..4);
```

$$\int_0^4 x^2\,dx$$

The problem is that if it were of the form

```
> Sum(x[i]^2* Delta[i],i=0..4);
```

$$\sum_{i=0}^{4} {x_i}^2\,\Delta_i$$

Then since there are only four terms given we could have x0 = 0. What are the other x's. We must have

```
> Delta[4]:=5/20;Delta[3]:=5/12;
```

$$\Delta_4 := \frac{1}{4}$$

$$\Delta_3 := \frac{5}{12}$$

Since we know that

```
> x[4]:=4;
```

$$x_4 := 4$$

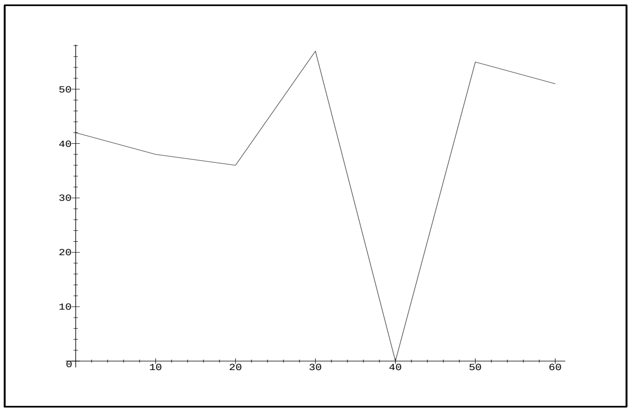
We must have that

```
> x[3]:=4-Delta[4];x[2]:=x[3]-Delta[3];
```

There we have the problem, 3.3 would have to be in the interval [x2,x3], but its not, x2=3.33333..

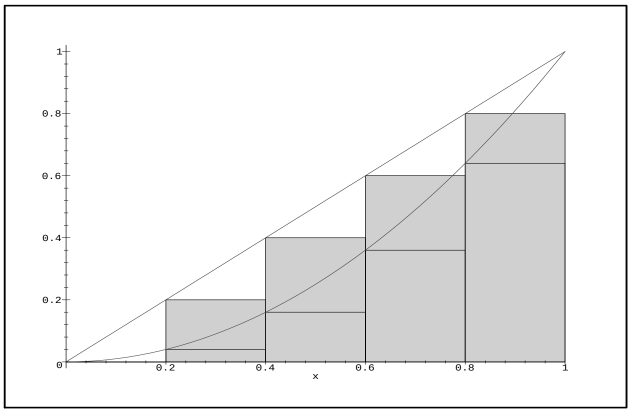
page 48 number 14

```
> plot([[0,42],[10,38],[20,36],[30,57],[40,0],[ 50,55],[60,51]]);
```



The problem asks for trapezoid, left, and midpoint approximations and asks which answer is most comvincing. One notes that the seven values given, connnected by a polygonal (straight line segments) graph divides the area into exactly six trapezoids which the trapezoid rule will calculate exactly. Thus the trapezoid approximation is surely the best. One should note that the function given is always positive. Thus its integral will be an increasing function. The graph of the integral function for the above polygonal graph should exhibit flex-points at 20,30,40,50, should be increasing, and should have a horizontal tangent at x=40.

```
> with(student):
> A:=leftbox(x^2,x=0..1,5):  B:=leftbox(x,x=0..1,5):
> plots[display]([A,B]);
```



From the picture it appears that the total area of the upper boxes is a little smaller than the area between the two curves - but its hard to tell.

```
> evalf(leftsum(x ,x=0..1,5)- leftsum(x^2,x=0..1,5));
```

$$.1600000000$$

```
> Int(x-x^2,x=0..1);
```

$$\int_0^1 x - x^2\,dx$$

```
> evalf(");
```

$$.1666666667$$

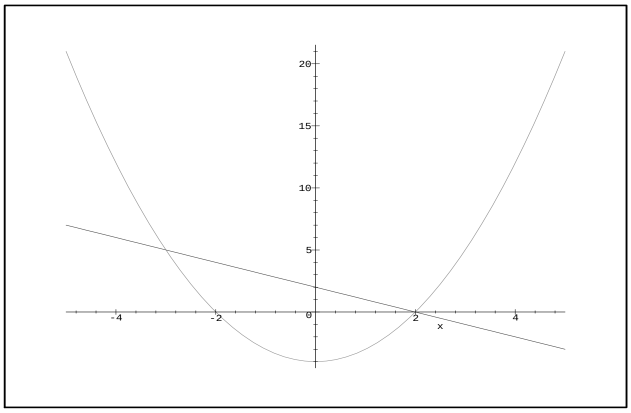
Hmmmm ... no wonder, the difference is only .0066..

page 56, number 7

This asks for the area between x=y^2-1 and y=2-x. One can, of course, solve for x in terms of y and integrate with respect to y. However one can also realize that interchanging x and y cannot change the answer since all this does is rotate it and flip it. Thus you can look at the area between

y=x^2-4, and x=2-y.

```
> plot({x^2-4,2-x},x=-5..5);
```




```
> solve( x^2-4=2-x,{x} );
```

$$\{x = -3\},\ \{x = 2\}$$

```
> Int((x^2-4) -(2-x),x=-3..3);
```

$$\int_{-3}^{3} x^2 - 6 + x\,dx$$

```
> int((x^2-4) -(2-x),x=-3..3);
```

$$-18$$