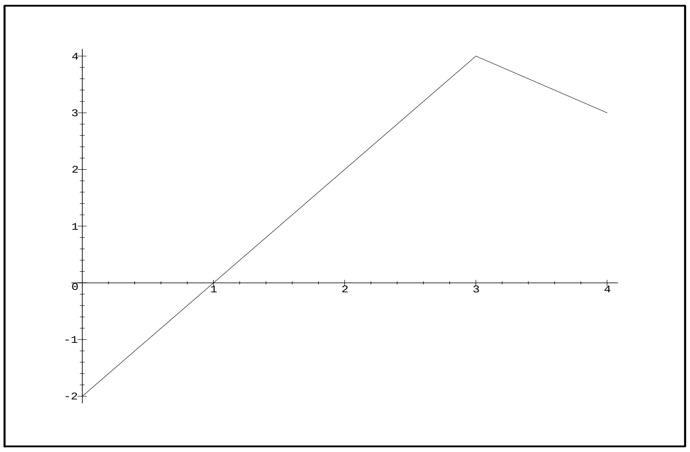
Practice Skills Test 1

Instructions: This is a practice exam for the first 'skills' test to be given on Tuesday, September 23. You can use any calculator up to, but not including, a TI-92. You must show your work.

1. Suppose $\int_1^5 f(x)\,dx = 1.2$, and $\int_1^5 g(x)\,dx = -3.1$. Evaluate the integral $\int_1^5 3\,x + 1 + 4\,f(x) - g(x)\,dx$.

2. Suppose the temperature rose linearly from 60 degrees to 80 degrees from 9 am to 12 am, and then stayed at 80 degrees for the next two hours. Calculate the average temperature over the 5 hour time interval.

3. Evaluate $\int_0^4 f(x)\,dx$, where $y = f(x)$ is the function graphed below.



4. Suppose f is a continuous function with $0 < \mathrm{f}(x)$ for $x < 5$ and $\mathrm{f}(x) < 0$ for $5 < x$.

Let $A_f(x) = \int_2^x \mathrm{f}(t)\,dt$. Make a rough sketch of the graph of $A_f$over the interval 0 to 8, showing where A_f is increasing and where it is decreasing.

5. Differentiate the following functions.

a) $\ln(a\,x + b)$

b) $\arcsin(x^2 + 1)$

c) $e^{(3\,x+1)}\sin(2\,x)$

6. Let $I = \int_0^5 \sqrt{x + 10}\,dx$. Write out the approximation R_5 to I. Is $I < R_5$? Why or why not?

7. The velocity of a particle moving on a straight line was measured at times $t = 0,\ 1,\ 3,\ 4$ (measured in seconds) and found to be respectively $v = 20,\ 10,\ -5,\ -10$. Estimate, on the information given, the average velocity and the average speed over the time interval from $t = 0..7$.

8. Find the area between the functions $y = x^2 - 4$ and $y = 4 - x^2$. Sketch the region.

9. Perform the following integrations:

a) $\displaystyle\int x \sin(x^2)\, dx$

b) $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\, dx$

c) $\displaystyle\int_1^5 \frac{x}{3\,x+1}\, dx$

d) $\displaystyle\int \frac{3\,x^3}{1+x^4}\, dx$

10. Given the error bound $\left| \displaystyle\int_a^b \mathrm{f}(x)\, dx - L_n \right| \leq \dfrac{K_1\,(b-a)^2}{2\,n}$ for the left hand rule.

Take $\mathrm{f}(x) = x^2$ and $a = 0$, $b = 10$. What value of n do we need in order that $\left| \displaystyle\int_0^1 x^2\, dx - L_n \right|$ be less or equal to 1.0?

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