Laboratory Calculus Worksheet 1
Thursday, August 29, 1996

Instructions: Work on the following problems during this laboratory period. Record the results of your group work in the space provided.

Date: ____________________

Team Members:

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1. Let g be the function shown below. Which of the following is the most reasonable estimate for $\int_1^2 g(x)dx$? -4, 4, 45, 450. Why?

2. Evaluate $\int_0^1 \sqrt{1-(x-1)^2}dx$

3. Suppose that f is a function such that $\int_0^3 f(x)dx = -1$.

   (a) Suppose that f is an *even* function. Explain why $\int_{-3}^3 f(x)dx = -2$.

   (b) If f is an *odd* function, what is the value of $\int_{-3}^3 f(x)dx$? Explain.

4. Show that $0 \le \int_0^\pi x \sin x dx \le \frac{\pi^2}{2}$.

5. Suppose that f is a continuous function. If the average value of f over the interval $[-3, 1]$ is 2 and the average value of f over the interval $[-3, 7]$ is 5, what is the average value of f over the interval $[1, 7]$?

6.

7.

8. Sketch the graph of a continuous function f such that $\int_{-2}^{3} f(t)dt = -10$ and

(a) $\int_{-2}^{3} |f(t)|dt = 10$

(b) $\int_{-2}^{3} |f(t)|dt \neq 10$

9.

10.

(a) $\int_0^2 f(x)dx$

(b) $\int_2^5 f(x)dx$

(c) $\int_0^5 f(x)dx$

(d) $\int_5^9 f(x)dx$

(e) $\int_0^{15} f(x)dx$

11. Explain why $\int_1^2 x^3 dx = \int_0^4 (x-2)^3 dx = \int_{-3}^{-2} (x+4)^3 dx$.

12.

13. (a) Use the racetrack principle to show that $1 + x \leq e^x \leq 1 + 3x$ when $0 \leq x \leq 1$.

(b) Use part (a) to find upper and lower bounds for $\int_0^1 e^x dx$.

14. (a) Show that $\pi(1 + \frac{1}{e}) \leq \int_0^{2\pi} e^{\sin x} dx \leq \pi(1 + e)$.

(b) Use the result in part (a) to find upper and lower bounds for $\int_0^{50\pi} e^{\sin x} dx$.

15. Suppose that f is a even function and continuous everywhere. Explain why the average value of f over the interval $[-a, a]$ is equal to the average value of f over the interval $[0, a]$.