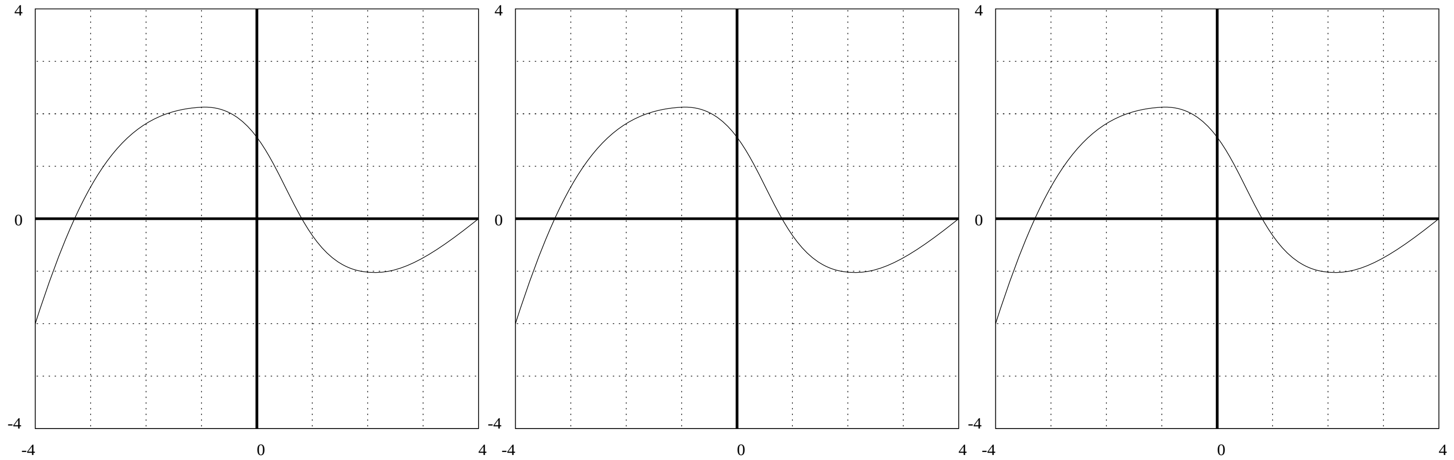
Laboratory Calculus Worksheet 3
Week of September 9, 1996
Individual Copy

Recall the following derivatives (they may come in handy):

Function	Derivative	Function	Derivative
$\arcsin u$	$\dfrac{1}{\sqrt{1-u^2}}$	$\operatorname{arccsc} u$	$-\dfrac{1}{u\sqrt{u^2-1}}$
$\arccos u$	$-\dfrac{1}{\sqrt{1-u^2}}$	$\operatorname{arcsec} u$	$\dfrac{1}{u\sqrt{u^2-1}}$
$\arctan u$	$\dfrac{1}{1+u^2}$	$\operatorname{arccot} u$	$-\dfrac{1}{1+u^2}$

1. Let g be the function graphed below. Estimate the value of $\displaystyle\int_{-4}^{4} g(x)dx$ by evaluating left, right, and midpoint sums with 4 equal subintervals, then draw sketches that illustrate these sums geometrically (one sketch on each copy of the graph below).



Graph of g — Graph of g — Graph of g

2. Evaluate $\displaystyle\lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}\left(\frac{j}{n}\right)^3$ by expressing it as a definite integral and then evaluating this integral using the Fundamental Theorem of Calculus.

3. For each of the following definite integrals, draw a sketch illustrating the approximations L_4, R_4, M_4, and T_4. Then compute each approximation and compare it with the exact value of the integral computed using the Fundamental Theorem of Calculus. How could you have used the sketches to predict the results of these comparisons?

   (a) $\displaystyle\int_{-2}^{2} x^2 dx$

   (b) $\displaystyle\int_{-\pi/2}^{\pi/2} \sin x dx$

4. Sketch the region bounded by the given curves, and find the area of that region exactly.

   (a) $y = \sin x, \; y = \cos x, \; x = 0, \; x = \pi$

   (b) $y = 9(4x+5)^{-1}, \; y = 2 - x$

5. Evaluate each of the following antiderivatives. Check your answers by differentiating.

   (a) $\displaystyle\int \frac{dx}{1+4x^2}$

   (b) $\displaystyle\int \frac{3}{4+x^2}$

6. Verify each antiderivative by differentiation.

   (a) $\displaystyle\int \sec x dx = \ln|\sec x + \tan x| + C$

   (b) $\displaystyle\int \arcsin x dx = x \arcsin x + \sqrt{1-x^2} + C$

   (c) $\displaystyle\int \arctan x dx = x \arctan x - \frac{1}{2}\ln\left(1+x^2\right) + C$

7. Recall that $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$, and that for *positive* x, $\dfrac{d}{dx}(\ln(x)) = \dfrac{1}{x}$. Show that $\dfrac{d}{dx}(\ln(|x|)) = \dfrac{1}{x}$ for all $x \neq 0$.