(1)

1.1 Real numbers

8/24/2005

* we need numbers to count, measure things, solve equations....

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$$

- $\mathbb{N}$: $1, 2, 3, 4, \dots$ natural #s
- $\mathbb{Z}$: $0, \pm 1, \pm 2, \pm 3, \dots$ integers
- $\mathbb{Q}$: $\frac{a}{b}$ $a, b \in \mathbb{Z}$, $b \neq 0$ rational #s
- $\mathbb{R}$: real #s ?

$\mathbb{R}$ includes all natural, integers, rational numbers as well as the irrational numbers such as π, e, $\sqrt{2}$, $\sqrt{3}$ etc....

* We can add and multiply real #s

$$a + b \quad , \quad a \cdot b$$

* The properties of these 2 operations are

$a + b = b + a$ for all $a, b \in \mathbb{R}$ (commutative

$a + (b + c) = (a + b) + c$ (associative

$a + 0 = a$ (additive identity)

$a + (-a) = 0$ ($\exists$ of opposite)

$ab = ba$ (commutative)

$a(bc) = (ab)c$ (associative)

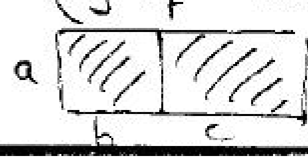
$a \cdot 1 = 1 \cdot a$ (multiplicative iden

$\frac{1}{a} = a^{-1}$ $\quad a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$ for $a \neq 0$ ($\exists$ of inverse)

distributive property

$a(b + c) = ab + ac$ $\longleftrightarrow$

$(a + b)c = ac + bc$



in terms of areas!

(2)

Example: by combining these properties

$(a+b)(c+d) = ?$

$= (a+b)c + (a+b)d = ac + bc + ad + bd$

* Properties of equality

If $a=b$ and c is any real THEN

$a+c = b+c \qquad ac = bc$

* Products involving 0 :

$a \cdot 0 = 0$ for all a

$ab = 0 \Longrightarrow$ either $a=0$ or $b=0$

* Properties of negatives :

$-(-a) = a \qquad (-a)b = -(ab) = a(-b)$

$(-a)(-b) = ab \qquad (-1)a = -a$

* Notation for reciprocals : $a \neq 0$: $a^{-1} = \frac{1}{a}$

$2^{-1} = \frac{1}{2} \qquad \left(\frac{3}{4}\right)^{-1} = \frac{1}{3/4} = \frac{4}{3}$

* Quotients : $\frac{a}{b} = \frac{c}{d} \rightsquigarrow ad = cb$

eg: $\frac{2}{3} = \frac{4}{6}$

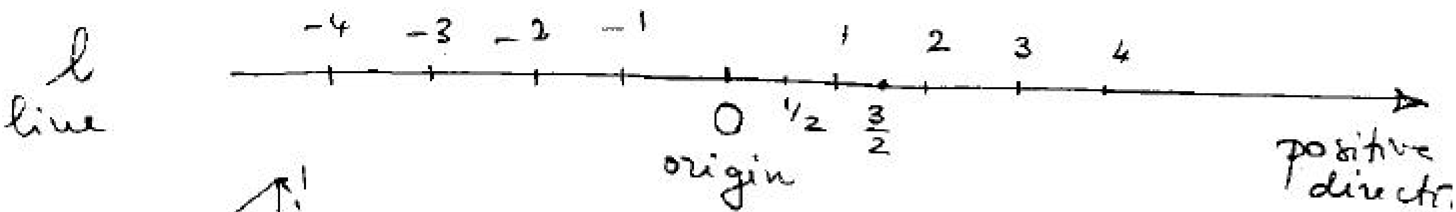
(3)

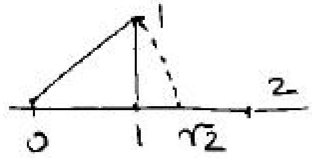
$$\frac{ad}{bd} = \frac{a}{b} \quad \text{for } d \neq 0$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd} \qquad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

etc...

* There is a one-to-one correspondence between points on a line and real numbers





this is called "Coordinate line" or real line

$$\begin{cases} a > b \iff a - b > 0 \\ a < b \iff a - b < 0 \end{cases}$$

Law of signs: $a, b > 0$ or $a, b < 0$

$\implies ab, \frac{a}{b} > 0$

$a > 0, b < 0 \implies ab, \frac{a}{b} < 0$

So: if $x < 0, y > 0$ THEN

$x^2 y > 0, \quad \frac{x}{y} + x < 0 \quad y(y - x) > 0$

(4)

Absolute value : $|a| = ?$

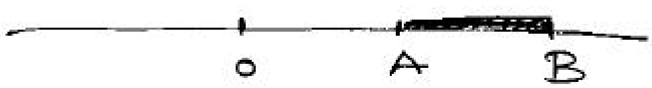
if $a \geq 0$ $|a| = a$, if $a < 0$ $|a| =$

$|-5| = 5 = -(-5)$ $\quad |-5| = |-1 \cdot 5| = |-1| \cdot |5$
$= 1 \cdot 5 = 5$

We need absolute value to measure the distance between two points on the real line :

Let A, B be 2 points on the real line with coordinates a, b respectively

$$\text{dist}(A, B) = |b-a| = |a-b| = \text{dist}(B,$$



$|-3-2| = 5$ $\quad |\pi - 4| = 4 - \pi$

$|x^2+4| = x^2+4$ $\quad |5-x| = ?$ if $x > 5$

$x > 5 \rightsquigarrow 0 > 5-x \rightsquigarrow |5-x| = -(5-x) = x$

Scientific form : $10^0 = 1$ $\quad 10^1 = 10$ $\quad 10^2 = 10$

$10^{-1} = \frac{1}{10}$ $\quad 10^{-2} = \frac{1}{100}$ etc... $\quad 20{,}700 = 2.07 \cdot 10^4$