MA 665 EXERCISES 3

(1) Let R be an integral domain with field of fractions K and let M be an R-module. Prove that the rank of M is equal to the dimension of the K-vector space $K \otimes_R M$.

(2) Prove that an $n \times n$ matrix A with entries in $\mathbb{C}$ satisfying $A^3 = A$ can be diagonalized. Is the same statement true over *any* field K?

(3) Determine the Jordan canonical form of the $n \times n$ matrix over $\mathbb{C}$ whose entries are all equal to 1.