

## MA 665 EXERCISES 5

- (1) Let  $R$  be a ring. Prove that every  $R$ -module is projective if and only if every  $R$ -module is injective.
- (2) Let  $R$  be a commutative ring. Prove that  $R[x]$  is a flat  $R$ -module.
- (3) Let  $M_1$  and  $M_2$  be  $R$ -modules. Show that  $M_1 \oplus M_2$  is an injective  $R$ -module if and only if both  $M_1$  and  $M_2$  are injective  $R$ -modules. Conclude that, if  $R$  is a PID that is not a field, then no nonzero finitely generated  $R$ -module is injective.