

MA 665 EXERCISES 7

- (1) Show that the following are equivalent.
- (a) B is an injective R -module.
 - (b) $\text{Ext}_R^i(A, B) = 0$ for all $i \neq 0$ and all A .
 - (c) $\text{Ext}_R^1(A, B) = 0$ for all A .
- (2) Let R be an integral domain with field of fractions K . Show that $\text{Tor}_1^R(K/R, B)$ is the torsion submodule of B for every R -module B .
- (3) Let m be an integer and d a divisor of m . Let $R = \mathbb{Z}/m\mathbb{Z}$. Show that

$$\cdots \xrightarrow{m/d} \mathbb{Z}/m\mathbb{Z} \xrightarrow{d} \mathbb{Z}/m\mathbb{Z} \xrightarrow{m/d} \mathbb{Z}/m\mathbb{Z} \xrightarrow{d} \mathbb{Z}/m\mathbb{Z} \rightarrow \mathbb{Z}/d\mathbb{Z} \rightarrow 0$$

is an infinite projective resolution of $\mathbb{Z}/d\mathbb{Z}$. Compute $\text{Tor}_n^R(\mathbb{Z}/d\mathbb{Z}, B)$ for all n and all B . (Note: the answer will depend on the parity of n .)