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Group Theory via Quilts

David Jensen and Kate Ponto

The UK Math Lab is the central hub for undergraduate engagement in mathematics at the University of Kentucky. It supports both undergraduate research projects and visualization projects, including sewing mathematical quilts.

The newest math quilt from the University of Kentucky Math Lab, pictured in Figure 1, is a multiplication table (also known as a Cayley table) for $SL(2,3)$, the group of 2×2 matrices of determinant 1, with entries in the field with 3 elements. This quilt is a companion piece to our previous quilt, pictured in Figure 2, which was a multiplication table for the symmetric group on 4 letters. In some ways, the two quilts look very similar – for example, both groups have order 24 – but in other ways, they are quite different.

A very visible difference between the quilts is the number of white squares along the main diagonal, from the top left to the bottom right. These correspond to elements whose square is the identity – that is, elements of order 1 or 2. The quilt in Figure 1 has only 2 white squares on the main diagonal, whereas the quilt in Figure 2 has many more. From this, we see that the two groups are not isomorphic.

This count of white squares in Figure 1 shows that in $SL(2,3)$ there is a unique 2×2 matrix of order 1 (the identity) and a unique 2×2 matrix of order 2 (-1 times the identity). The subgroup generated by these two matrices is normal. If you stand far away from the quilt, like in Figure 3, you

Figure 1

Figure 2

Figure 3

Figure 4

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Figure 5

can imagine that the 2×2 blocks are a single square. In this way, Figure 3 is a multiplication table for the quotient by this normal subgroup. The quotient, $\mathrm{PSL}(2,3)$, happens to be isomorphic to the alternating group A_4. This isomorphism is visible in the two quilts. Indeed, the pattern in Figure 3 matches that of the top left 12×12 block of Figure 2, pictured in Figure 4.

Similarly, the 8×8 block of yellow and orange squares in the upper left corner of Figure 1 is another normal subgroup. The other 8×8 blocks, one consisting of the pink and purple squares, and the other consisting of the blue and green squares, are its cosets. Again, if you stand far away, like in Figure 5, you can imagine that the 8×8 blocks are each a single square. This shows the order 8 subgroup of $\mathrm{SL}(2,3)$ is normal, and the quotient is $\mathbb{Z}/3\mathbb{Z}$.

If you try to do the same thing with the 4×4 block of yellow (and white!) squares, everything gets all mixed up. This is easiest to see in the right columns where the yellow squares are mixed in with the orange squares. While this is not a proof, it suggests (correctly) that this subgroup is not normal.

So, from this quilt we can see that $\mathrm{SL}(2,3)$ has normal subgroups of order 2 and 8. In the same way, in the S_4 quilt we can see normal subgroups of order 12 and 4.

These quilts represent significant effort of both undergraduate student and faculty members of the University of Kentucky Math Lab. They are hand sewn. We made this choice for a couple reasons – it helps us achieve the necessary precision and, more importantly, allows many people to participate simultaneously. This coordinated participation gives the students ample opportunity to interact with faculty and each other. We view this interaction as one of the primary benefits of these projects.

You can find more information about our other quilts and details on how we make the quilts at the UK Math Lab website ukmathlab.blogspot.com.

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