MA 114 Worksheet #27: Differential equations

- 1. (a) Is $y = \sin(3x) + 2e^{4x}$ a solution to the differential equation $y'' + 9y = 50e^{4x}$? Explain why or why not.
 - (b) Explain why every solution of $dy/dx = y^2 + 6$ must be an increasing function.
 - (c) What does is mean to say that a differential equation is linear or nonlinear?
- 2. Find all values of α so that $y(x) = e^{\alpha x}$ is a solution of the differential equation y'' + y' 12y = 0.
- 3. A tank has pure water flowing into it at 10 liters/min. The contents of the tank are kept thoroughly mixed, and the contents flow out at 10 liters/min. Salt is added to the tank at the rate of 0.1 kg/min. Initially, the tank contains 10 kg of salt in 100 liters of water. Formulate an initial value problem (that is, a differential equation along with initial conditions) whose solution is the quantity of salt in the tank at any time t. Do not solve the initial value problem.
- 4. Consider a tank with 200 liters of salt-water solution. A salt-water solution, with a concentration of 2 grams per liter, pours into the tank at a rate of 4 liters per minute. The well-mixed solution in the tank pours out at the same rate of 4 liters/minute. Write a differential equation expressing the rate of change in the concentration, c(t), of salt in the tank. Do not solve.