1 Prove that $f : \mathbb{R} - \{0\} \longrightarrow \mathbb{R} - \{0\}$, $f(x) = 1/x$, is continuous.

2 Let X be a Hausdorff space in which every subset is compact. Prove that X is finite.

3 Let X and Y be compact metric spaces and let $\{f_n : X \longrightarrow Y\}_{n=1}^{\infty}$ be a collection of continuous surjections for which the sequence (f_n) converges uniformly to a function $f : X \longrightarrow Y$. Prove that f must be surjective.

4 Let X be a connected Hausdorff space and let $\tilde{X}$ be its one-point compactification. Prove that $\tilde{X}$ is connected if and only if X is not compact.

5 Let $\mathbb{R}^{\omega} = \mathbb{R} \times \mathbb{R} \times \cdots$ have the product topology and let $\pi_i : \mathbb{R}^{\omega} \longrightarrow \mathbb{R}$ be the usual projection map onto the ith factor. Let (x_n) be a sequence of points in $\mathbb{R}^{\omega}$ such that each sequence $(\pi_i(x_n))$ converges to $a_i \in \mathbb{R}$. Prove that (x_n) converges to the point $a = (a_1, a_2, \ldots)$.

6 Prove that a retract of a simply connected space is simply connected.

7 Prove that the complement of a finite set in $\mathbb{R}^3$ is simply connected.

8 Let X be a compact Hausdorff space and let $\{A_n\}_{n=1}^{\infty}$ be a collection of closed sets for which each A_n has empty interior. Prove that $\cup_{n=1}^{\infty} A_n$ has empty interior. (This is a special case of the Baire Category Theorem.)