DEPARTMENT OF MATHEMATICS

TOPOLOGY PRELIMINARY EXAMINATION, JANUARY 9, 2017

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1 Consider the rotation action of S^1 on R^2. Show that the orbit space R^2/S^1 by this action is homeomorphic to $[0, \infty)$.

2 Let X be a topological space. Suppose for each pair (x, K) of a point $x \in X$ and a closed set K so that $x \notin K$, there are disjoint open sets U and V so that $x \in U$ and $K \subset V$.

Prove that the closure of a compact set in X is compact.

3 A T_1 space X is $T_{3\frac{1}{2}}$ if for every $x \in X$ and open set U containing x, there is a continuous function

$$\chi : X \longrightarrow I$$

so that $\chi(x) = 0$ and $\chi \equiv 1$ on $X - U$. Prove that the product of $T_{3\frac{1}{2}}$ spaces is $T_{3\frac{1}{2}}$.

4 Let M be a connected n-manifold.

(a) Prove that M is path connected.
(b) If $n \geq 2$ and $F \subset M$ is a finite set, prove that $M - F$ is path connected.

5 Let $X = S^2 \cup (\{(0,0)\} \times [-1, 1])$ be the union of the sphere and a diameter, and let $Y = X \vee S^1$. Compute the fundamental group of Y.

6 There are spaces with nonabelian fundamental groups. Use this fact to show that the fundamental group of $S^1 \vee S^1$ is nonabelian. (This problem does not ask you to compute $\pi_1(S^1 \vee S^1)$ and this computation will not be considered a solution.)

7 Let $p : E \longrightarrow B$ be a covering map. Suppose p has a right inverse s, i.e. a continuous map $s : B \longrightarrow E$ such that $p \circ s = \mathrm{id}_B$. Prove that if E is path connected, then p is a homeomorphism.

8 Let X be a connected graph and Y be a connected subgraph. Show that the map

$$\pi_1(Y) \longrightarrow \pi_1(X)$$

induced by inclusion is injective.