# DEPARTMENT OF MATHEMATICS

## Topology Preliminary Examination
## January 9, 2018

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1 Let $T$ be the cofinite topology on $\mathbf{Z}$. Show that the sequence $\{1, 2, 3, \ldots\}$ converges to every point of $(\mathbf{Z}, T)$. What are the convergent sequences in $(\mathbf{Z}, T)$?

2 Let $p : X \longrightarrow Y$ be a continuous open surjection, and let $R \subset X \times X$ be defined by

$$R = \{(x_1, x_2) \in X \times X \mid p(x_1) = p(x_2)\}.$$

Show that $Y$ is Hausdorff if and only if $R \subset X \times X$ is closed.

3 Suppose $G$ is a topological group, and let $C$ be the connected component of the identity element $e \in G$. Show that $C$ is a normal subgroup of $G$ (normal in the sense of group theory, meaning that for all $g \in G$, we have $gCg^{-1} \subset C$).

4 Let $X = \mathcal{C}(S^1, S^1)$ be the space of continuous functions $S^1 \longrightarrow S^1$ with the compact-open topology. Is $X$ compact?

5 Prove that $S^1 \times \{x_0\}$ is a retract of $S^1 \times S^1$ but that it is not a deformation retract of $S^1 \times S^1$ for any $x_0 \in S^1$.

6 Let $F$ be a four-point subset of the boundary $S^1$ of the unit disk $D^2$. Compute $\pi_1(D^2/F)$.

7 Prove that there is no map $f : S^n \longrightarrow S^1$ for $n \geq 2$ such that $f(-x) = -f(x)$ for all $x \in S^n$.

8 Find all two-sheeted coverings of the Klein bottle up to equivalence.