1 Let $f : X \longrightarrow Y$ be a surjection, where X is Hausdorff and Y is arbitrary. Assume that for each open cover $\{U\}$ of X there is an open cover $\{V\}$ of Y such that $\{f^{-1}(V)\}$ refines $\{U\}$. Prove that f is one-to-one.

2 Let (X, d) be a metric space and assume that for every descending sequence $A_1 \supset A_2 \supset \cdots$ of non-empty closed sets whose diameters $\text{diam}A_n \to 0$, the intersection $\cap_{n=1}^{\infty} A_n \neq \varnothing$. Prove that (X, d) is complete.

3 Let X be a metric space. Prove that a set $A \subset X$ is open if and only if for every compact $C \subset X$, $A \cap C$ is open in C.

4 (a) Give an example of a continuous surjection $f : X \longrightarrow Y$ for which X is locally compact and Y is not locally compact.
(b) Let X be locally compact and $r : X \longrightarrow A$ be a retraction. Prove that A is locally compact.

5 Let $f : (0, 1) \longrightarrow (0, 1)$ be a continuous increasing function (i.e. $x \leq y \Rightarrow f(x) \leq f(y)$). Prove that f is uniformly continuous.

6 Let X be normal and let $X = U_1 \cup U_2$, where U_1 and U_2 are open. Prove that there exist continuous functions $f_i : X \longrightarrow [0, 1], i = 1, 2$, such that $f_1(x) + f_2(x) = 1$, for all $x \in X$, and $f_i(y) = 0$ for all $y \in X - U_i$.

7 Let $A \subset \mathbb{R}^2$ be a bounded non-empty set with interior $\text{Int}A$ and boundary $\text{Bd}A$. If $(0, 0) \in \text{Int}A$, prove that there is a point $(x, 0) \in \mathbb{R}^2$ for which $[0, x) \times \{0\} \subset \text{Int}A$ and $(x, 0) \in \text{Bd}A$.

8 Prove that the fundamental group of S^2 is trivial.

9 Let $p : E \longrightarrow B$ be a covering map. Suppose that p has a section, i.e. there is a continuous map $\sigma : B \longrightarrow E$ such that $p \circ \sigma = \text{id}_B$. Let $e \in E$ and $b = p(e)$.

(a) Prove that
$$p_* : \pi_1(E, e) \longrightarrow \pi_1(B, b)$$
is an isomorphism.
(b) Prove that if E is path connected, then p is a homeomorphism.

10 One of the fundamental results of Topology II is the following: $\pi_1(S^1)$ is isomorphic to $\mathbb{Z}$. Give a precise description of this isomorphism. Be as complete as time permits.