1 Define "embedding". Give two spaces X and Y such that X embeds in Y and Y embeds in X, but X and Y are not homeomorphic.

2 (a) Sketch a homeomorphism between $[0,1] \times [0,1)$ and $(0,1) \times [0,1)$.
(b) If $X \times W$ and $Y \times W$ are homeomorphic, must X and Y be homeomorphic?

3 Let $X, (Y_1, d_1), (Y_2, d_2)$ be metric spaces and let D be the metric on $Y_1 \times Y_2$ defined by

$$D\left((y_1, y_2), (y_1', y_2')\right) = \left(d_1(y_1, y_1')^2 + d_2(y_2, y_2')^2\right)^{1/2}.$$

Prove that a function $f : X \longrightarrow Y_1 \times Y_2$ is uniformly continuous if and only if $\pi_i \circ f : X \longrightarrow Y_i$ is uniformly continuous, $i = 1, 2$ ($\pi_i : Y_1 \times Y_2 \longrightarrow Y_i$ is the canonical projection).

4 Let X be a compact metric space and let A_1, A_2, A_3 be closed sets in X such that $A_1 \cap A_2 \cap A_3 = \varnothing$. Prove that there exists a $\delta > 0$ which satisfies the following property: If $A \subset X$ has diameter $< \delta$, then there is an i for which $A \cap A_i = \varnothing$.

5 Let

$$T = \{(x, y) \in \mathbb{R}^2 \mid x, y \geq 0, x + y \leq 1\}.$$

Show that if A is a closed subset of a normal space X and $f : A \longrightarrow T$ is a continuous map, then f can be extended to a continuous map $F : X \longrightarrow T$.

6 A topological space X is said to have the *fixed point property* if each continuous map $f : X \longrightarrow X$ has a fixed point. Let (X, d) be a compact metric space and for all $n \geq 1$ let $A_n \subset X$ satisfy the property that there exists a continuous function $f_n : X \longrightarrow A_n$ for which $d(f_n(x), x) < 1/n$ for all $x \in X$. Prove that if each A_n has the fixed point property, then so does X.

7 Let $H \subset \mathbb{R}^n$ be an $(n-2)$-dimensional subspace. Determine the fundamental group of $\mathbb{R}^n - H$.

8 Let X be a topological space, let $\mathcal{C}(I, X)$ be the space of continuous maps from the interval $I = [0,1]$ to X with the compact-open topology, and let $A \subset \mathcal{C}(I, X)$ be the subspace of constant maps. Show

(a) A is homeomorphic to X.
(b) A is a deformation retract of $\mathcal{C}(I, X)$. [Hint: Consider the map $(f, t) \mapsto g$, where $f \in \mathcal{C}(I, X)$, $t \in I$, and $g(s) = f(ts)$.]

9 Let $p : E \longrightarrow B$ be a covering map, let Y be path connected and locally path connected, and let $f, g : Y \longrightarrow B$ be continuous functions which are homotopic. Prove that if f can be lifted to a map $\tilde{f} : Y \longrightarrow E$, then g can be similarly lifted.